Random Hypersurface Models for Extended Object Tracking

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***Abstract*—Target tracking algorithms usually assume that the received measurements stem from a point source. However, in many scenarios this assumption is not feasible so that measurements may stem from different locations, named measurement sources, on the target surface. Then, it is necessary to incorporate the target extent into the estimation procedure in order to obtain robust and precise estimation results. This paper introduces the novel concept of Random Hypersurface Models for extended targets. A Random Hypersurface Model assumes that each measurement source is an element of a randomly generated hypersurface. The applicability of this approach is demonstrated by means of an elliptic target shape. In this case, a Random Hypersurface Model specifies the random (relative) Mahalanobis distance of a measurement source to the center of the target object. As a consequence, good estimation results can be obtained even if the true target shape significantly differs from the modeled shape. Additionally, Random Hypersurface Models are computationally tractable with standard nonlinear stochastic state estimators.**

***Index Terms*—Tracking, extended objects, state estimation, random sets**

I. INTRODUCTION

In most tracking algorithms, the received measurements are assumed to originate from a point source without an extent. However, there are several situations in which this assumption is not valid. For instance, modern high-resolution radar devices may receive measurements from different scattering centers, called measurement sources, on the extended target. An illustration of such a scenario is given in Fig. 1. As a consequence, tracking algorithms have to estimate the target extent in addition to the target position in order to improve the robustness and precision of the estimation results. A major difficulty is that the measurement sources are unknown. Usually, even the target shape itself is unknown. This kind of problem often occurs in military surveillance with radar devices [1], [2], but can also be frequently found in many other areas like robotics. An example is the tracking of humans with a robot-borne laser range scanner [3]. A second important scenario (see Fig. 1) is tracking a collectively moving group of point targets [2]. If the point targets move closely together compared to the sensor resolution, it becomes hard to tackle the data association problem. In this case, it is suitable to consider the group of point targets as one single extended object, since there is a high interdependency between single measurements. Possible application areas include the tracking of plane clouds, vehicle convoys [2], [4] or flocks of birds.

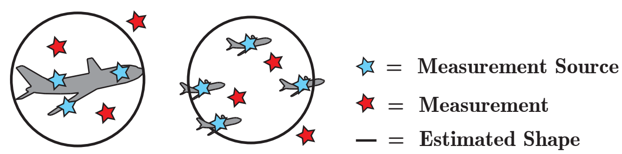


Fig. 1: Extended target (left) and group target (right).

A. Problem Formulation

We treat the problem of tracking the position and shape of an unknown extended object in a plane based on noisy position measurements. At each time step k, a finite set of two-dimensional position measurements $\{\underline{\hat{z}}_{k,l}\}_{l=1}^{n_k}$ may be available (see Fig. 1). Each individual measurement $\underline{\hat{z}}_{k,l}$ is the noisy observation of a two-dimensional point $\underline{\tilde{z}}_{k,l}$, named measurement source, which is known to lie on the target surface, i.e.,

$$\underline{\hat{z}}_{k,l} = \underline{\tilde{z}}_{k,l} + \underline{\boldsymbol{w}}_{k,l} \ , \tag{1}$$

where $\underline{\boldsymbol{w}}_{k,l}$ denotes additive white observation noise[1]. The probability distribution of the measurement noise $\underline{\boldsymbol{w}}_{k,l}$ is assumed to be known since it results from the particular sensor model, e.g., a radar device. However, the location of the measurement source $\underline{\tilde{z}}_{k,l}$ is totally unknown.

The goal is to estimate the position and shape of the extended target object. Since the true shape of the extended object is unknown, one typically approximates the true shape by means of a basic geometric shape like an ellipsoid or a rectangle. Then, one wants to estimate the parameters of this geometric shape. The temporal evolution of the extended object is modeled by means of a so-called *extended motion model* that captures both the kinematics and the change of shape of the target object (details are given in Section III-C).

A generic model of the generation process of one measurement is illustrated in Fig. 2. In general, it consists of two steps: For a given geometric object, first a measurement source is generated. The measurement itself is then a noisy observation of the measurement source according to (1). Existing approaches for extended object tracking mainly differ in modeling the target geometry. This is reflected by the measurement source model (see the first component in Fig. 2). The measurement model, i.e., the second component

[1]Note that all random variables are printed bold face in this paper.

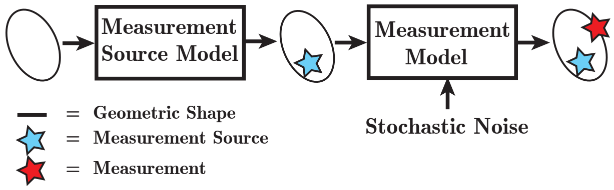


Fig. 2: Generation process of measurements.

in Fig. 2, is assumed to be given by (1), since it results from the particular sensor model.

B. State of the Art

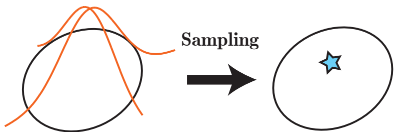
There exists a variety of approaches for incorporating the target extent into target tracking algorithms (for a detailed overview see [1], [5]). Approaches based on modeling particular features of the target explicitly are proposed in [6], [7], [8], [9]. For instance, in [6] the motion of the extended object is modeled as one bulk that is characterized by a finite set of individual components (like points on the object). Each of these individual components is a potential measurement source. Hence, it is necessary to cope with data association problems. Here, in this work, we focus on approaches that model measurement sources implicitly and thus avoid data association problems. These are in particular spatial distribution target models and set-theoretic target models discussed in the following.

1) Spatial Distribution Models: In [10], [11], the target shape is modeled by means of a so-called spatial distribution. Each measurement source is an independent random draw from a two-dimensional probability distribution. This approach is illustrated in Fig. 3a for an elliptic shape and a Gaussian spatial distribution (red-colored function). Spatial distribution models mainly suffer from two disadvantages: First, in real world applications it is nearly impossible to determine a reasonable spatial distribution. Since the target object, including the properties and shape of the target surface is unknown, it is unpredictable which measurement source is responsible for a particular measurement. Spatial distribution models are not able to incorporate this lack of knowledge such that a more or less reasonable probability distribution for the measurement sources has to be guessed. Second, the statistical properties of the spatial distribution in general depend on the parameter vector of the target shape. For instance, the wider the shape of the target object, the larger is the variance of the spatial distribution. From a Bayesian point of view, a spatial distribution model can therefore also be seen as a hierarchical probability model. Such hierarchical probability models usually require a high computational effort. When the complexity of the shape grows, the complexity of the representation of the spatial distribution grows dramatically. Furthermore, closed-form solutions typically do not exist so that Monte Carlo methods are often used to obtain an approximated solution. However, in the case of elliptical target models, [2] provides a closed-form solution in case there is no measurement error. There, an elliptic object extension is modeled with a random matrix that is treated as an additional state variable. In case the measurement error is not negligible, the problem can only be tackled [4], [12] with further assumptions.

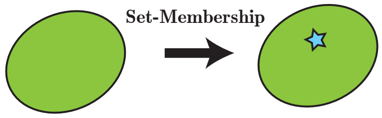
2) Set-Theoretic Models: A novel approach for extended object tracking based on combined set-theoretic and stochastic fusion was recently proposed in [13] for circular discs and in [14] for rectangles. This approach only requires the measurement sources to be on the target surface. No further (statistical) assumptions about the measurement sources are made. An illustration is given in Fig. 3b. The set of possible measurement sources is drawn in green. In case there is no measurement noise, the problem can be formulated as a set-theoretic estimation procedure. Stochastic measurement noise then requires a combined set-theoretic and stochastic estimator. This approach is able to deal with unknown target objects and noisy measurements in a systematic manner. At the moment circular discs and axis-aligned rectangles can be handled. Furthermore, it is necessary to assume that the number of measurements that are received at a particular time step depends on the size of the extended object. This assumption is justified in most real world applications, nonetheless it is the major restriction of the approach.

II. Key Idea and Contributions

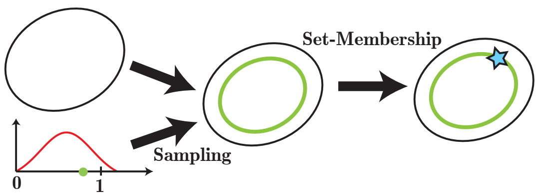
In this paper, we introduce a novel measurement source model (see Fig. 2) for extended targets called *Random Hypersurface Model*. A *Random Hypersurface Model* specifies the generation of *one* measurement source in two steps. For a given extended target shape, first a hypersurface is generated randomly. The measurement source is then selected from this hypersurface (according to an arbitrary, unknown rule). An intuitive illustration of a particular *Random Hypersurface Model* for elliptical targets is given in Fig. 3c. For a given elliptical target shape, the generated hypersurface is a scaled version of the bound of the ellipsoid. The scaling factor is specified by a one-dimensional probability density function (see the red-colored function in Fig. 3c). The random scaling factor can be interpreted as the (relative) distance of the measurement source from the target center with respect to the Mahalanobis distance induced by the true ellipsoid. The statistics of this scaling factor can be chosen independently of the target shape. With this approach, it is avoided to deal with hierarchical probability densities such that a standard nonlinear stochastic state estimator can be used for inference. Furthermore, rather soft restrictions on the measurement sources on the target object are made. Therefore, this approach is more robust to systematic errors in the target model, i.e., when the true shape and distribution of the measurement sources do not coincide with the modeled target shape. Finally, it is not necessary to assume that the size of the extended object depends on the number of received measurements as in [13], [14]. As a consequence, the size of the target object can be estimated even if only one measurement per time step is available. A further major advantage is that arbitrary target shapes can be modeled since the target shape only depends on the deterministic shape function that can easily be exchanged.



(a) Spatial Distribution Model.



(b) Set-Theoretic Model.



(c) Random Hypersurface Model.

Fig. 3: Modeling extended targets: Approaches.

In the following section, we first introduce a so-called *Random Set Model* for extended targets. We show that *Random Set Models* are an extension of spatial distribution models [10], set-theoretic models [13], and also *Random Hypersurface Models*. As a consequence, it is possible to treat these three approaches in a consistent manner. Subsequently, in Section III-B, a Bayesian filter for *Random Set Models* is derived, which allows to estimate the target extent and position based on the received measurements. In Section IV, the novel special case, named *Random Hypersurface Model*, which is highly relevant for practical applications, is presented in detail. A particular *Random Hypersurface Model* for elliptical target shapes is then derived in Section IV-C. The applicability of this model is demonstrated by means of a group target tracking example in Section V.

III. Random Set Model for Extended Objects

We are now going to introduce a so-called *Random Set Model* for the measurement source model (see Fig. 2). Random sets allow for capturing both set-valued and stochastic uncertainties simultaneously, which is in particular suitable to model extended targets. This approach allows a consistent treatment of spatial distribution models, set-theoretic models, and *Random Hypersurface Models*, which are introduced in the next section. A random set [15] is a random experiment whose outcomes are sets. In particular, we only treat sets that can be described by a finite number of parameters. In this case, the statistics of the random set are determined by the statistics of the parameters.

A. Generative Measurement Source Model

We assume that the shape of the target object can be described at each time step k by a set of the form

$$\mathcal{O}(\underline{p}_k) := \{\underline{z} | \underline{z} \in \mathbb{R}^2 \text{ and } g(\underline{z}, \underline{p}_k) \leq 0\}$$

in which $\underline{p}_k$ is a parameter vector and $g(\underline{z}, \underline{p}_k)$ is the geometric shape function.

Example 1. A circular target object is specified by the parameter vector $\underline{p}_k := \left[x_k^c, y_k^c, r_k\right]^T$, where $\left[x_k^c, y_k^c\right]^T$ is the center and r_k the radius. With $\underline{z} = \left[z_1, z_2\right]^T$, the shape function is given by $g(\underline{z}, \underline{p}_k) := (x_k^c - z_1)^2 + (y_k^c - z_2)^2 - r_k^2$.

Next, we are going to explain how *one* measurement source $\underline{\tilde{z}}_{k,l}$ is modeled to be generated from a given extended object $\mathcal{O}(\underline{p}_k)$. This generative model for a measurement source is based on two steps: First, a so-called *measurement set* is generated randomly. The measurement source itself is then selected from this measurement set. This selection does not have to follow any (statistical) rule. It is only known that the measurement source is an element of the measurement set. Note that this set-valued uncertainty is fundamentally different from a stochastic uncertainty.

A formal definition is given as follows: A *Random Set Model* is a conditional probability density function of the form

$$f(\underline{p}_{k,l}^m \,|\, \underline{p}_k)$$

in which $\underline{p}_{k,l}^m$ is the parameter of a set

$$\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m) := \{\underline{z} \,|\, \underline{z} \in \mathbb{R}^2 \text{ and } \mathbf{C}(\underline{z}, \underline{\boldsymbol{p}}_{k,l}^m)\} \ ,$$

where $\mathbf{C}(\underline{z}, \underline{\boldsymbol{p}}_{k,l}^m)$ denotes a constraint that specifies the measurement solution set. For a given target shape $\mathcal{O}(\underline{p}_k)$, first a measurement set $\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m)$ is generated randomly. The measurement source $\underline{\tilde{z}}_{k,l}$ is then selected (according to an unknown, arbitrary rule) from the set $\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m)$, i.e.,

$$\underline{\tilde{z}}_{k,l} \in \mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m) \ ,$$

which is equivalent to $\mathbf{C}(\underline{\tilde{z}}_{k,l}, \underline{\boldsymbol{p}}_{k,l}^m)$.

Random Set Models are a generalization of two already existing approaches for modeling extended targets:

- *Spatial Distribution Model*
  If there are no set-valued uncertainties, a spatial distribution model [10], [2] is obtained. In other words: A spatial distribution is a *Random Set Model* where $\underline{\boldsymbol{p}}_{k,l}^m$ is two-dimensional and the measurement set is a singleton set $\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m) := \{\underline{\boldsymbol{p}}_{k,l}^m\}$ that results from the constraint $\mathbf{C}(\underline{z}, \underline{\boldsymbol{p}}_{k,l}^m) := (\underline{\tilde{z}}_{k,l} - \underline{\boldsymbol{p}}_{k,l}^m = 0)$.
- *Set-Theoretic Model*
  If the *Random Set Model* is deterministic, i.e., $f(\underline{\boldsymbol{p}}_{k,l}^m \,|\, \underline{\boldsymbol{p}}_{k,l}) = \delta(\underline{\boldsymbol{p}}_{k,l}^m - \underline{\boldsymbol{p}}_{k,l})$ with Dirac distribution $\delta(\cdot)$, and the measurement set coincides with the extended object, i.e., $\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m) := \mathcal{O}(\underline{\boldsymbol{p}}_{k,l}^m)$, a set-theoretic target model [13], [14] is obtained.

It is important to note that the measurement $\underline{\hat{z}}_{k,l}$ itself is a noisy observation of the measurement source $\underline{\tilde{z}}_{k,l}$ according to the measurement model (1).

B. Backward Inference

In the following, we are going to derive a formal Bayesian filter for the parameter vector $\underline{p}_k$ of the extended object $\mathcal{O}(\underline{p}_k)$

at time step k given a *Random Set Model*. For this purpose, the uncertainty about the current parameters of the extended object at time step k is captured by a random variable $\underline{\boldsymbol{p}}_k$ with prior probability density function $f_l^p(\underline{p}_k)$. The knowledge about $\tilde{\underline{z}}_{k,l}$ is given by the random vector $\hat{\underline{z}}_{k,l} - \underline{\boldsymbol{w}}_{k,l}$. Hence, with $\underline{\boldsymbol{z}}_{k,l} := \hat{\underline{z}}_{k,l} - \underline{\boldsymbol{w}}_{k,l}$ we obtain the constraint $\mathbf{C}(\underline{\boldsymbol{z}}_{k,l}, \underline{\boldsymbol{p}}_{k,l}^m)$, which must be satisfied. This constraint can be interpreted as a Boolean random variable $\boldsymbol{c}_{k,l} \in \{\text{true}, \text{false}\}$, which is a function of $\underline{\boldsymbol{z}}_{k,l}$ and $\underline{\boldsymbol{p}}_{k,l}^m$ according to $\boldsymbol{c}_{k,l} := \mathbf{C}(\underline{\boldsymbol{z}}_{k,l}, \underline{\boldsymbol{p}}_{k,l}^m)$. As a consequence, the posterior probability density function is given by

$$f_l^e(\underline{p}_k) := f_l^p(\underline{p}_k \mid \boldsymbol{c}_{k,l} = \text{true}) \ .$$

In general, the computation of $f_l^e(\underline{p}_k)$ can be performed by considering the joint probability density function

$$f_l(\underline{p}_k, c_{k,l}, \underline{z}_{k,l}, \underline{p}_{k,l}^m) := f_l^p(\underline{p}_k) \cdot f(\underline{z}_{k,l}) \cdot f(\underline{p}_{k,l}^m | \underline{p}_k) \cdot \delta_{c_{k,l} = \mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m)} \ , \quad (2)$$

where $\delta_{c_{k,l} = \mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m)} = 1$ if $c_{k,l} = \mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m)$ and 0 otherwise. Bayes' theorem followed by a marginalization then leads to

$$f_l^e(\underline{p}_k) = a_{k,l} \cdot \int\int f_l(\underline{p}_k, \text{true}, \underline{z}_{k,l}, \underline{p}_{k,l}^m) d\underline{z}_{k,l} d\underline{p}_{k,l}^m \ , \quad (3)$$

where $a_{k,l}$ is a normalization constant. This is equivalent to truncating values that do not satisfy the constraint, i.e.,

$$f_l^c(\underline{p}_k, \underline{z}_{k,l}, \underline{p}_{k,l}^m) := \begin{cases} a_{k,l} \cdot f_l(\underline{p}_k, \underline{z}_{k,l}, \underline{p}_{k,l}^m) & \text{if } \mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

with a subsequent marginalization

$$f_l^e(\underline{p}_k) = \int\int f_l^c(\underline{p}_k, \underline{z}_{k,l}, \underline{p}_{k,l}^m) d\underline{z}_{k,l} d\underline{p}_{k,l}^m \ .$$

The practical computation of $f_l^e(\underline{p}_k)$ depends highly on the particular form of the constraint $\mathbf{C}(\underline{\boldsymbol{z}}_k, \underline{\boldsymbol{p}}_{k,l}^m)$. In general, it is not possible to compute $f_l^e(\underline{p}_k)$ in closed form. However, approximation techniques exist for several types of constraints. In the following, we discuss the above mentioned special cases:

- *Spatial Distribution Model*
  In case of a spatial distribution model we obtain $f_l^e(\underline{p}_k) = f_l^p(\underline{p}_k \mid \underline{\boldsymbol{p}}_{k,l}^m + \underline{\boldsymbol{w}}_{k,l} = \hat{\underline{z}}_{k,l})$. For details see [10].
- *Set-Theoretic Model*
  For a set-theoretic model, a given measurement yields in a nonlinear inequality constraint for the prior density

  $$f_l^c(\underline{p}_k, \underline{z}_{k,l}) := a_{k,l} \cdot \begin{cases} f_l(\underline{p}_k, \underline{z}_{k,l}) & \text{if } \mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}) \\ 0 & \text{otherwise} \end{cases}$$

  with a subsequent marginalization

  $$f_l^e(\underline{p}_k) = \int f_l^c(\underline{p}_k, \underline{z}_{k,l}) d\underline{z}_{k,l} \ .$$

  Note that the above inference mechanism for set-theoretic target models has not been investigated so far. The combined set-theoretic and stochastic estimators introduced in [13], [14] do *not* make use of prior knowledge given by a prior density $f_l^p(\underline{p}_k)$. As a consequence, random sets that capture the uncertainties about $\underline{p}_k$ are obtained.

In case $\mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m)$ is a (nonlinear) equality constraint, (4) can be seen as a constrained state estimation problem for which many different solution techniques exists. Possible approaches include a reformulation of (4) as a pseudo-measurement [16] or the fusion procedure of Dirac-mixture densities in [17]. In case $\mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m)$ is a (nonlinear) inequality constraint, also a variety of proper techniques for computing (4) exists in literature [16]. The favorable method depends highly on the particular form of the constraint.

C. Extended Motion Models

Next, we treat the dynamic aspects of extended object tracking based on *Random Set Models*. For the sake of simplicity, we assume that the target state vector only consists of the shape parameter vector $\underline{p}_k$. In real world applications, the target vector would also include information about the current velocity and acceleration, for instance. The state vector $\underline{\boldsymbol{p}}_k$ is assumed to evolve according to a known Markov model given by the conditional density function $f(\underline{p}_k | \underline{p}_{k-1})$. The predicted probability density at time step k results from the Chapman-Kolomogorov equation

$$f^p(\underline{p}_k) = \int f(\underline{p}_k | \underline{p}_{k-1}) f^e(\underline{p}_{k-1}) d\underline{p}_{k-1} \ ,$$

where $f^e(\underline{p}_{k-1})$ is the posterior density at time step $k-1$.

The predicted probability density can then be updated with the set of measurements $\{\hat{\underline{z}}_{k,l}\}_{l=1}^{n_k}$ as described in Section III-B. If we assume that the measurement noise $\underline{\boldsymbol{w}}_{k,l}$ is white, the measurements can be processed sequentially according to

$$\begin{aligned} f_0^e(\underline{p}_k) &:= f^p(\underline{p}_k) \\ f_l^e(\underline{p}_k) &:= a_{k,l} \cdot f_{l-1}^p(\underline{p}_k) \cdot f(c_{l,k} = \text{true} \mid \underline{p}_k) \\ &= f_{l-1}^p(\underline{p}_k) \cdot \int\int f(\underline{z}_{k,l}) \cdot f(\underline{p}_{k,l}^m | \underline{p}_k) \cdot \\ &\quad \delta_{\text{true} = \mathbf{C}(\underline{z}_{k,l}, \underline{p}_{k,l}^m)} d\underline{z}_{k,l} d\underline{p}_{k,l}^m \end{aligned}$$

and setting $f^e(\underline{p}_k) := f_{n_k}^e(\underline{p}_k)$.

IV. Random Hypersurface Model

We now introduce a novel approach for modeling extended target objects called *Random Hypersurface Model*, which is a *Random Set Model* whose measurement sets are hypersurfaces. In contrast to a spatial distribution that determines a two-dimensional probability distribution over the measurement sources, and the set-theoretic approach that does not make any statistical assumptions, *Random Hypersurface Model* are based on a one-dimensional distribution over hypersurfaces.

A. Generative Measurement Source Model

A *Random Hypersurface Model* is specified by the mapping

$$\underline{\boldsymbol{p}}_{k,l}^m = h(\underline{\boldsymbol{p}}_k, \boldsymbol{s}_{k,l}) \ , \quad (5)$$

where $\boldsymbol{s}_k$ is a one-dimensional random variable and $h(\cdot,\cdot)$ is a function such that the measurement set $\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m)$ is a hypersurface, i.e., a one-dimensional sub-manifold in two-dimensional space. The *Random Hypersurface Model* is then given by the conditional probability density function

$$\begin{aligned} f(\underline{p}_{k,l}^m \mid \underline{p}_k) &= \int f(\underline{p}_{k,l}^m \mid \underline{p}_k, s_{k,l}) f(s_{k,l}) ds_{k,l} \\ &= \int \delta(\underline{p}_{k,l}^m - h(\underline{p}_k, s_{k,l})) f(s_{k,l}) ds_{k,l} \ . \end{aligned}$$

An important special case is obtained if the hypersurfaces are scaled versions of the bound of the target object. Therefore, let $\underline{p}_k := \left[\underline{m}_k^T, \underline{r}_k^T\right]^T$, where $\underline{m}_k^T$ is the location of the target object and $\underline{r}_k^T$ defines the shape of the object in the sense that

$$\begin{aligned} \mathcal{O}(\left[\underline{m}_k^T, \underline{r}_k^T\right]^T) &:= \{\underline{z} | \underline{z} \in \mathbb{R}^2 \text{ and } g^*(\underline{z} - \underline{m}_k, \underline{r}_k) \leq 0\} \ , \\ \overline{\mathcal{O}}(\left[\underline{m}_k^T, \underline{r}_k^T\right]^T) &:= \{\underline{z} | \underline{z} \in \mathbb{R}^2 \text{ and } g^*(\underline{z} - \underline{m}_k, \underline{r}_k) = 0\} \ , \end{aligned}$$

with $g(\underline{z}, \underline{p}_k) = g^*(\underline{z} - \underline{m}_k, \underline{r}_k)$. The measurement set is a scaled version of the bound of the target object if

$$\mathcal{M}(\underline{\boldsymbol{p}}_{k,l}^m) = \underline{m}_k + \boldsymbol{s}_{k,l} \cdot \overline{\mathcal{O}}(\left[\underline{0}^T, \underline{r}_k^T\right]^T) \ .$$

The scaling factor is specified by the random variable $\boldsymbol{s}_{k,l}$. Note that the center of the target object is not changed. An intuitive interpretation of such a *Random Hypersurface Model* is that it specifies the (relative) distance of the measurement source from the target center. The underlying distance measure is induced by the geometric shape function, for instance Euclidean norm (circular objects) or Mahalanobis distance (elliptical objects). Furthermore, *Random Hypersurface Models* are adequate to model any star-convex object. In practical applications, one typically restricts the scaling factor $s_{k,l}$ to the interval $[0, 1]$, since the measurement sources lie on the surface of the target object. Note that the statistics of the scaling factor $\boldsymbol{s}_{k,l}$ do not (have to) depend on $\underline{p}_k$. So this is not a hierarchical probability model. Furthermore, since only the relative distance is specified, it is not specified where the measurement source lies exactly. The measurement source is only known to be an element of the hypersurface. This is a suitable way to express the lack of knowledge about the true target object.

B. Backward Inference

Bayesian backward inference for *Random Hypersurface Models* can be performed with standard nonlinear stochastic state estimation techniques. The joint probability density $f_l(\underline{p}_k, \underline{z}_{k,l}, \underline{p}_{k,l}^m)$ in (4) can be computed by means of the prediction step of a nonlinear state estimator, since $\underline{\boldsymbol{p}}_{k,l}^m$ in (5) is the result of a (nonlinear) transformation of $\underline{\boldsymbol{p}}_{k,l}$ and $\boldsymbol{s}_{k,l}$. The density $f_l^c(\underline{p}_k, \underline{z}_{k,l}, \underline{p}_{k,l}^m)$ is then the result of enforcing the constraint

$$g(\underline{z}_{k,l}, \underline{p}_{k,l}^m) = 0 \ . \tag{6}$$

This can be performed by employing a nonlinear stochastic state estimator like [18], [19] and considering the equality constraint as a pseudo-measurement, i.e., a fictitious measurement 0 with variance 0. Apart from this, many elaborate techniques exist for dealing with nonlinear state equality constraints (see for instance [17]). The subsequent marginalization depends highly on the chosen density representation but in general this does not pose any technical difficulties.

It is important to note that with an increasing number of measurements, the probability density of $\underline{p}_k$ approaches the true parameters of the extended object. Hence, it is possible to estimate the target extend even if only one measurement is available per time step.

C. Example: Elliptic Targets

Below, a *Random Hypersurface Model* for elliptic target shapes is presented. The generated hypersurface is a scaled version of the bound of the true ellipsoid (see Fig. 3c). The scaling factor then specifies the relative Mahalanobis distance from the measurement source to the target center. Elliptical shapes are highly relevant for real world applications since many target objects like ships can be considered approximately as an ellipsoid. Furthermore, elliptic shapes supply orientation information, which is quite useful in real world applications.

Definition 1 (Ellipse)**.** A two-dimensional ellipse with center $\underline{c}_k$ and positive semi-definite shape matrix $\mathbf{A}_k$ is given by the set $\{\underline{z} \mid \underline{z} \in \mathbb{R}^2 \text{ and } (\underline{z} - \underline{c}_k)^T A_k^{-1} (\underline{z} - \underline{c}_k) \leq 1\}$.

In order to avoid the treatment of random positive semi-definite matrices, we employ directly the Cholesky decomposition of $\mathbf{A}_k$ given by $\mathbf{L}_k \mathbf{L}_k^T$, where $\mathbf{L}_k := \begin{bmatrix} l_k^{(1)} & 0 \\ l_k^{(3)} & l_k^{(2)} \end{bmatrix}$ is a lower triangular matrix with positive diagonal entries. The parameter vector of an ellipsoid is then given by $\underline{p}_k = \left[\underline{c}_k^T, l_k^{(1)}, l_k^{(2)}, l_k^{(3)}\right]^T$, which consists of the center and the non-zero entries of the Cholesky decomposition. The shape function $g(\underline{z}, \underline{p}_k)$ is then given by

$$g(\underline{z}, \underline{p}_k) := (\underline{z} - \underline{c}_k)^T (\mathbf{L}_k \cdot \mathbf{L}_k^T)^{-1} (\underline{z} - \underline{c}_k) - 1 \ .$$

The nonlinear function $h(\underline{\boldsymbol{p}}_{k,l}, \boldsymbol{s}_{k,l})$ is assumed to be

$$h(\underline{\boldsymbol{p}}_{k,l}, \boldsymbol{s}_{k,l}) = \left[\underline{c}_k, \boldsymbol{s}_{k,l} \cdot l_k^{(1)}, \boldsymbol{s}_{k,l} \cdot l_k^{(2)}, \boldsymbol{s}_{k,l} \cdot l_k^{(3)}\right]^T \ ,$$

which corresponds to a scaling of the ellipsoid while leaving the center and orientation unchanged.

V. Experiments

We demonstrate the applicability of *Random Hypersurface Models* for ellipsoid target shapes by means of a group target tracking example (see Fig. 4). For the first 8 time steps, the point targets are arranged in a fixed relative position, then they start to change their relative positions. In the simulation, all involved densities are assumed to be Gaussian and we employ the unscented transformation [19] for inference by interpreting the constraint in (6) as a pseudo-measurement. The position of the group is known to evolve according to a linear motion model $\underline{\boldsymbol{p}}_k = \underline{\boldsymbol{p}}_{k-1} + \underline{\hat{u}}_{k-1} + \underline{\boldsymbol{v}}_{k-1}$, where $\underline{\boldsymbol{p}}_k = \left[\underline{\boldsymbol{c}}_k^T, \boldsymbol{l}_k^{(1)}, \boldsymbol{l}_k^{(2)}, \boldsymbol{l}_k^{(3)}\right]^T$ is the state vector, which

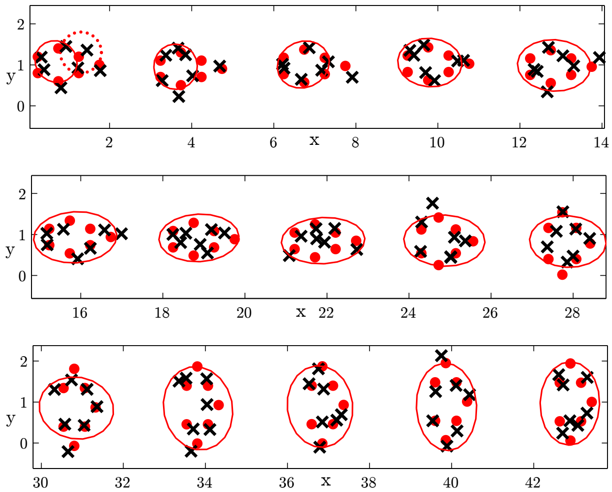


Fig. 4: Tracking a group of point targets: Snippets of the state space. Point targets (red dots), measurements (crosses), estimated ellipsoid (red) plotted for several time steps. The (expectation) of the prior ellipsoid is plotted with red dots.

consists of the center and extent of the group. The random vector $\underline{v}_{k-1}$ is unbiased Gaussian noise with covariances $\text{diag}(0.001, 0.005, 0.005, 0.005, 0.005)$. The vector $\hat{\underline{u}}_{k-1}$ denotes deterministic system input. Note that the uncertainty of the target shape is increased at each time step in order to capture shape changes. At each time step, one position measurement according to (1) is obtained from each group member. The random vector $\underline{\boldsymbol{w}}_{k,l}$ models zero-mean Gaussian noise with covariance $\text{diag}(0.025, 0.025)$. The Gaussian density of the scaling factor $\boldsymbol{s}_{k,l}$ in (5) has an expectation of 0.8 and a variance 0.015. We made use of prior density for $\underline{\boldsymbol{p}}_k$ with covariance matrix $\text{diag}(0.4, 0.4, 0.2, 0.2, 0.2)$. Fig. 4 shows the state space of the simulation for the first time steps. It can be seen that the shape of the target group is approached quite fast, although the prior shape is quite different. Also, the deformation of the group (after the eighth time step) is captured immediately. Hence, *Random Hypersurface Models* are in general capable of tracking a group of point targets. Note that employing pseudo-measurements with the unscented transformation [19] for constraint enforcing is quite an ad-hoc solution. More elaborate techniques tailored to the particular constraint will yield even better results.

VI. Conclusions and Future Work

In this paper, we have presented the theoretical concept of *Random Set Models* for extended targets, which is an extension of the two existing techniques spatial distribution models and set-theoretic target models. Second, a novel and promising special case called *Random Hypersurface Models* has been proposed. The applicability of this approach was shown by means of elliptic shapes, which can be treated with a standard stochastic state estimator.

Future work consists of investigating *Random Hypersurface Models* for other target shapes. An interesting problem would be to estimate an arbitrary target shape. Furthermore, it must be evaluated which state estimation technique yields the best estimation results for *Random Hypersurface Models*. For a real world application, it is also necessary to consider multiple models (for instance for the dynamics), false measurements, and track associations.

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