Association-free Tracking of Two Closely Spaced Targets

Marcus Baum and Uwe D. Hanebeck

***Abstract*— This paper introduces a new concept for tracking closely spaced targets in Cartesian space based on position measurements corrupted with additive Gaussian noise. The basic idea is to select a special state representation that eliminates the target identity and avoids the explicit evaluation of association probabilities. One major advantage of this approach is that the resulting likelihood function for this special problem is unimodal. Hence, it is especially suitable for closely spaced targets. The resulting estimation problem can be tackled with a standard nonlinear estimator. In this work, we focus on two targets in two-dimensional Cartesian space. The Cartesian coordinates of the targets are represented by means of extreme values, i.e., minima and maxima. Simulation results demonstrate the feasibility of the new approach.**

I. INTRODUCTION

Multi-target tracking [1] is a fundamental problem that arises in many application areas. For instance, in air surveillance, groups of airplanes are tracked by means of radars. One of the major challenges in multi-target tracking is that it is unknown from which target a particular measurement stems. In order to tackle this problem, elaborate data association techniques have been developed in the past [1].

This paper deals with the problem of tracking two coordinated point targets. This problem occurs for instance, when two airplanes are flying in a formation. In case that the measurement noise is high in comparison to the relative distance of the targets, it becomes difficult to distinguish between the two targets. A similar problem occurs when point features on an extended object are to be tracked [2], [3], [4].

In this paper, we introduce a novel approach to group target tracking [4], [5], [6] called *Unique State filter*. The basic idea is to select a state representation in such a way that the association problem disappears and the target identity is elimated. For this purpose, we represent the Cartesian coordinates of the two targets by means of extreme values, i.e., minima and maxima. This representation ensures that there is a unique state that specfies the target positions (when the target identity is ignored).

The remainder of this paper is structured as follows: After a detailed problem formulation in Section II, we briefly state related methods for data association (Section III). In Section IV, the new approach called *Unique State filter* is introduced. Simulation results in Section VI then show the feasibility of the new data association method. Finally, the conclusions and an outlook to future work are given in Section VII.

M. Baum and U. D. Hanebeck are with the Intelligent Sensor-Actuator-Systems Laboratory (ISAS), Institute for Anthropomatics, Karlsruhe Institute of Technology (KIT), Germany. `marcus.baum@kit.edu`, `uwe.hanebeck@ieee.org`

II. PROBLEM FORMULATION

In this paper, we consider the problem of tracking the Cartesian coordinates of two point targets in two-dimensional space. We assume that at each time step exactly two measurements, namely one from each target, is available. These measurements are noise-corrupted observations of the target positions. The measurement-to-target association is unknown.

More formally, at each time step, the positions of the two targets $\underline{z}_{1,k} = [x_{1,k}, y_{1,k}]^T \in \mathbb{R}^2$ and $\underline{z}_{2,k} = [x_{2,k}, y_{2,k}]^T \in \mathbb{R}^2$ shall be tracked. At each time step k, two position measurements $\hat{\underline{y}}_{1,k} \in \mathbb{R}^2$ and $\hat{\underline{y}}_{2,k} \in \mathbb{R}^2$ with Gaussian noise are received, i.e.,

$$\begin{aligned} \hat{\underline{y}}_{\pi(1),k} &= \underline{z}_{1,k} + \underline{\boldsymbol{v}}_{1,k} \ , \text{ and} \\ \hat{\underline{y}}_{\pi(2),k} &= \underline{z}_{2,k} + \underline{\boldsymbol{v}}_{2,k} \ , \end{aligned} \tag{1}$$

where $\pi : \{1,2\} \to \{1,2\}$ is the target-to-measurement assignment. The random variables $\underline{\boldsymbol{v}}_{1,k}$ and $\underline{\boldsymbol{v}}_{2,k}$ denote Gaussian measurement noise, both with identical diagonal covariance matrices $\mathrm{diag}([c^x_{v,k}, c^y_{v,k}])$.

In this paper, we assume that the targets are moving collectively in the sense that the target positions evolve according to the linear motion models

$$\begin{aligned} \underline{z}_{1,k+1} &= \underline{z}_{1,k} + \hat{\underline{d}}_k + \underline{\boldsymbol{w}}_{1,k} \ , \text{ and} \\ \underline{z}_{2,k+1} &= \underline{z}_{2,k} + \hat{\underline{d}}_k + \underline{\boldsymbol{w}}_{2,k} \ , \end{aligned} \tag{2}$$

with common system input $\hat{\underline{d}}_k$ and individual system noises $\underline{\boldsymbol{w}}_{1,k}$ and $\underline{\boldsymbol{w}}_{2,k}$, which are identically distributed according to a zero-mean Gaussian distribution with covariance $\mathrm{diag}([c^x_{w,k}, c^y_{w,k}])$. This motion model captures the group behavior while allowing changes in the relative position of the two targets. However, the proposed method in this paper is not limited to this motion model. In practical applications, the system input $\hat{\underline{d}}_k$ is typically unknown and has to be estimated based on the kinematics of the group [5], [7] . The introduced method can easily be extended for this purpose.

In this work, we focus on scenarios in which the targets are closely spaced compared to the measurement noise, i.e., the validation gates of the predicted measurements overlap highly. Note that in this paper, we are only interested in the positions of the two targets. The target identity, i.e., the track labels are not desired.

III. RELATED WORK

The considered problem can in general be tackled with a probabilistic data association method. In this section, we give a brief overview of data association techniques suitable for the problem. In the following, we roughly distinguish *implicit* and *explicit* data association techniques. A data association method is called explicit, if it explicitly computes the probability of feasible measurement-to-target assignments. On the other hand, an implicit method does not evaluate measurement-to-target assignments directly. In contrast to the method proposed in this paper, the mentioned data association techniques are in general not restricted to the above problem formulation. They are also capable of dealing with clutter and larger number of targets, for instance.

A. Explicit Data Association

The solution of the exact Bayesian formulation of the data association problem is in general intractable due to the increasing complexity of the probability densities and the exponentially growing number of association hypotheses. A variety of different approaches has been developed in order to obtain approximate solutions. A simple method for data association is the so-called *Nearest Neighbor Filter* [8] that assigns each observation to the most probable target. Since this hard decision may be wrong, this filter provides poor results in case of closely spaced targets. A popular data association technique is the *Joint Probabilistic Data Association Filter* (JPDAF) [9]. The JPDAF can be seen as an approximation of the exact Bayesian solution of the data association problem. JPDAF performs a weighted update of all target states in the validation gate according to the association probabilities. In order to deal with increasing complexity, the JPDAF ignores correlations between the target states and removes resulting multimodalities in the target state by means of analytic moment matching. An adjusted JPDAF algorithm for estimating an unordered set of targets was proposed in [10].

During recent years, Monte Carlo methods [11], [12] for approximating the exact Bayesian formulation have been developed. The so-called Multiple Hypothesis Tracker (MHT) [13] maintains all feasible association hypotheses over several time steps. Since the number of feasible association hypotheses grows exponentially over time, reduction methods like pruning and Gaussian mixture reduction are required.

The so-called Probabilistic Multiple Hypothesis Tracker (PMHT) [14] assumes that each single association is independently generated according to a discrete stochastic process. This formulation yields an incomplete data problem that can be solved by means of Expectation Maximization (EM).

B. Implicit Data Association

In the context of finite random sets, the so-called Finite Set Statistics (FISST) [15] allows for formulating an optimal multi-target Bayes filter. However, since the FISST filter recursion is in general intractable, the PHD-Filter [16] provides a tractable alternative by propagating the first order statistical moment of a finite random set.

The basic idea of the Symmetric Measurement Equation (SME) filter [17], [18], [19] is to define a symmetric measurement equation which maps the state of the targets to a pseudo-measurement constructed from the real measurements. Due to the symmetry of the measurement equation, it is not necessary to explicitly calculate the measurement-to-target association. A symmetric measurement equation is typically constructed by means of symmetric polynomials [18], [19]. The *SME filter* transforms the data association problem into a nonlinear state estimation problem with non-additive Gaussian noise. The first works on the *SME filter* employed the Extended Kalman filter (EKF) [20] for dealing with the nonlinear measurement equation. Recently, it was proposed to employ the Unscented Kalman filter (UKF) [18], [19] for the *SME filter*.

In [21], the memorylessness of the exponential distribution is exploited for avoiding explicit data association in the context of topic intensity tracking.

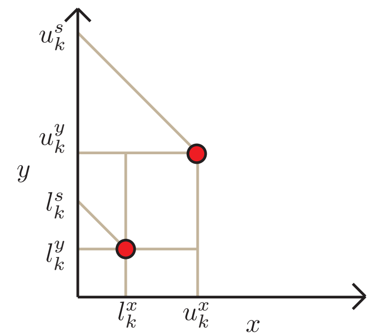


Fig. 1: Representation of the Cartesian coordinates of two targets by means of extreme values.

IV. THE UNIQUE STATE FILTER

The basic idea behind the *Unique State filter* is to select a representation of the two target positions that is invariant to permutation of the target positions, and yields a measurement equation that does not directly require for evaluating assignment hypotheses.

In the following, we introduce such a representation of the two target positions $\underline{z}_{1,k}$ and $\underline{z}_{2,k}$ that fulfills the above conditions. For this purpose, we first restrict ourselves to a one-dimensional version of the problem, where two one-dimensional target positions $x_{1,k}$ and $x_{2,k}$ on the x-axis are to be estimated based on the measurements $\hat{x}_{1,k}$ and $\hat{x}_{2,k}$. If we directly estimate the Cartesian coordinates $x_{1,k}$ and $x_{2,k}$, we always have to make a case distinction whether $\hat{x}_{1,k}$ or $\hat{x}_{2,k}$ stems from $x_{1,k}$ or $x_{2,k}$. As a consequence, the resulting likelihood function is bimodal. Hence, it is often not suitable to represent the state with a Gaussian distribution, especially for closely spaced targets. This is even the case with the *SME filter*, which does not explicitly compute the two possible associations.

The basic idea of the *Unique State filter* is to select a representation of the two targets that is unique with respect to permuations of the components of the transformed state vector. For instance, such a representation can be given by

$$\begin{aligned} l_k^x &= \min\{x_{1,k}, x_{2,k}\} \text{ and} \\ u_k^x &= \max\{x_{1,k}, x_{2,k}\} \ . \end{aligned}$$

The measurement update and prediction also has to be performed in the transformed space, i.e., l_k^x and u_k^x are to be estimated by means of the pseudo measurements $\min\{\hat{x}_{1,k}, \hat{x}_{2,k}\}$ and $\max\{\hat{x}_{1,k}, \hat{x}_{2,k}\}$. Since l_k^x and u_k^x are uniquely defined by means of the pseudo measurements, the resulting likelihood function for the transformed problem is unimodal.

Remark 1: One property of the *Unique State filter* is that the target identity gets lost. We do not know which of the two targets $x_{1,k}$ or $x_{2,k}$ coincides with l_k^x. This is also the case for the PHD-filter, where the target identity is not directly available.

An extension of the above approach to two-dimensional state space is not straightforward. This is due to the fact that two Cartesian points are not uniquely specified by their extreme values on the axes (see Fig. 1). Hence, further information has to be incorporated in the state vector. One possible solution is to estimate additionally

$$\min\{x_{1,k} + y_{1,k}, x_{2,k} + y_{2,k}\} \ .$$

All together we employ the following state vector for two targets in two-dimensional space (see Fig. 1)

$$\begin{aligned} \underline{x}_k &= \begin{bmatrix} l_k^x \\ u_k^x \\ l_k^y \\ u_k^y \\ l_k^s \\ u_k^s \end{bmatrix} = \begin{bmatrix} \min\{x_{1,k}, x_{2,k}\} \\ \max\{x_{1,k}, x_{2,k}\} \\ \min\{y_{1,k}, y_{2,k}\} \\ \max\{y_{1,k}, y_{2,k}\} \\ \min\{x_{1,k} + y_{1,k}, x_{2,k} + y_{2,k}\} \\ \max\{x_{1,k} + y_{1,k}, x_{2,k} + y_{2,k}\} \end{bmatrix} \\ &= T(\underline{z}_{1,k}, \underline{z}_{2,k}) \ , \end{aligned} \tag{3}$$

where

- l_k^x, u_k^x, l_k^y, and u_k^y are the extreme values on the x- and y-axis,
- l_k^s and u_k^s serve for uniquely defining the two target positions. For positive target coordinates, l_k^s and u_k^s are the minimal and maximal Manhattan distance from the origin.

For the two target positions $\underline{z}_{1,k}$ and $\underline{z}_{2,k}$, the state vector $\underline{x}_k$ is uniquely defined. For given $\underline{x}_k$, the two target positions turn out to be

$$\left[l_k^x, l_k^y\right]^T \text{ and } \left[u_k^x, u_k^y\right]^T$$

if $\min\{l_k^x + l_k^y, u_k^x + u_k^y\} = l_k^s$ and otherwise

$$\left[l_k^x, u_k^y\right]^T \text{ and } \left[u_k^x, l_k^y\right]^T \ .$$

Note that it is not possible to reconstruct the target identity, i.e., it is left open which of these points is $\underline{z}_{1,k}$ or $\underline{z}_{2,k}$.

Remark 2: The vector $\underline{x}_k$ is not a state according to the definition used in classical control theory, because it is not the smallest possible subset of system variables that represent the systems state at any given time. Similar, a quaternion is a non-minimal description of a three-dimensional rotation.

The goal is to estimate $\underline{x}_k = T(\underline{z}_{1,k}, \underline{z}_{2,k})$ based on measurements of $T(\hat{\underline{y}}_{1,k}, \hat{\underline{y}}_{2,k}) = T(\underline{z}_{1,k} + \underline{\boldsymbol{v}}_{1,k}, \underline{z}_{2,k} + \underline{\boldsymbol{v}}_{2,k})$. For this purpose, we derive an estimator that recursively computes the probability density of $\underline{\boldsymbol{x}}_k$ given the accumulated measurements

$$p(\underline{x}_k | \hat{\underline{y}}_{1:k}) \ ,$$

where $\hat{\underline{y}}_{1:k} := \{\hat{\underline{y}}_1, \ldots, \hat{\underline{y}}_k\}$ and pseudo measurement $\hat{\underline{y}}_i = T(\hat{\underline{y}}_{1,k}, \hat{\underline{y}}_{2,k})$. We assume that this probability density is Gaussian, i.e., $p(\underline{x}_k | \hat{\underline{y}}_{1:k}) = \mathcal{N}(\hat{\underline{x}}_k^e, \mathbf{C}_k^e)$. The prediction for the state at time step k is denoted with

$$p(\underline{x}_k | \hat{\underline{y}}_{1:k-1}) = \mathcal{N}(\hat{\underline{x}}_k^p, \mathbf{C}_k^p) \ .$$

In the following, we first describe the measurement update step, which takes the prediction and the next position measurements of the two targets in order to compute the updated estimate. Subsequently, we treat the state prediction.

A. Measurement Update Step

In order to perform a measurement update, we have to derive a measurement equation, which relates the state $\underline{\boldsymbol{x}}_k$ to the pseudo measurement $\hat{y}_k$ constructed from the individual position measurements $\hat{\underline{y}}_{1,k}$ and $\hat{\underline{y}}_{2,k}$.

A linear measurement equation, can be derived with the help of the following approximation

$$\begin{aligned} \hat{y}_k &= T(\underline{z}_{1,k} + \underline{\boldsymbol{v}}_{2,k}, \underline{z}_{2,k} + \underline{\boldsymbol{v}}_{2,k}) \\ &= T(\underline{z}_{1,k}, \underline{z}_{2,k}) - \\ &\quad T(\underline{z}_{1,k}, \underline{z}_{2,k}) + T(\underline{z}_{1,k} + \underline{\boldsymbol{v}}_{2,k}, \underline{z}_{2,k} + \underline{\boldsymbol{v}}_{2,k}) \\ &\approx \underline{x}_k - \\ &\quad \hat{\underline{x}}_k^p + T(\underline{z}_{1,k} + \underline{\boldsymbol{v}}_{2,k}, \underline{z}_{2,k} + \underline{\boldsymbol{v}}_{2,k}) \ . \end{aligned}$$

Hence, the linear measurement equation

$$\hat{\underline{y}}_k = \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{\delta}}_k \tag{4}$$

with noise term $\underline{\boldsymbol{\delta}}_k := -\hat{\underline{x}}_k^p + T(\underline{z}_{1,k} + \underline{\boldsymbol{v}}_{2,k}, \underline{z}_{2,k} + \underline{\boldsymbol{v}}_{2,k}$ is obtained. The noise term can be approximated without reconstructing $\underline{z}_{1,k}$ and $\underline{z}_{2,k}$ by means of the following approximation

$$\underline{\boldsymbol{\delta}}_k := -\hat{\underline{x}}_k^p + \begin{bmatrix} \min\left\{\hat{l}_k^x + \boldsymbol{v}_{1,k}^x, \hat{u}_k^x + \boldsymbol{v}_{2,k}^x\right\} \\ \max\left\{\hat{l}_k^x + \boldsymbol{v}_{1,k}^x, \hat{u}_k^x + \boldsymbol{v}_{2,k}^x\right\} \\ \min\left\{\hat{l}_k^y + \boldsymbol{v}_{1,k}^y, \hat{u}_k^y + \boldsymbol{v}_{2,k}^y\right\} \\ \max\left\{\hat{l}_k^y + \boldsymbol{v}_{1,k}^y, \hat{u}_k^y + \boldsymbol{v}_{2,k}^y\right\} \\ \min\left\{\hat{l}_k^s + \boldsymbol{v}_{1,k}^s, \hat{u}_k^s + \boldsymbol{v}_{2,k}^s\right\} \\ \max\left\{\hat{l}_k^s + \boldsymbol{v}_{1,k}^s, \hat{u}_k^s + \boldsymbol{v}_{2,k}^s\right\} \end{bmatrix} ,$$

where

- $\hat{l}_k^*$ and $\hat{u}_k^*$ with $* \in \{x, y, s\}$ are the current estimates, i.e., the predicted values, for l_k^* and u_k^*, and
- $\boldsymbol{v}_k^x$ and $\boldsymbol{v}_k^x$ models uncorrelated Gaussian noise with variance $c_{v,k}^x$ and $c_{v,k}^y$, and
- $\boldsymbol{v}_k^s$ is Gaussian noise with variance $c_{v,k}^x + c_{v,k}^y$ that is uncorrelated to $\boldsymbol{v}_k^x$ and $\boldsymbol{v}_k^y$.

The statistics of the noise term can in general be approximated with a Gaussian distribution by deriving analytic expressions [22] or by employing a nonlinear state estimator

[20] like the Unscented Kalman Filter (UKF) [23] or the Gaussian Filter [24]. Since the maximum of two Gaussian distributed random variables with similar variance is almost again Gaussian distributed, a Gaussian assumed density filter provides good results.

Given the predicted probability density for the parameters at time step k

$$p(\underline{x}_k|\underline{\hat{y}}_{1:k-1}) = \mathcal{N}(\underline{\hat{x}}_k^p, \mathbf{C}_k^p) \ ,$$

the updated estimate $p(\underline{x}_k|\underline{\hat{y}}_{1:k})$ according to the measurement model (4) is also Gaussian with mean $\underline{\hat{x}}_k^e$ and covariance $\mathbf{C}_k^e$ and results from the Kalman filter equations [20]

$$\begin{aligned}\underline{\hat{x}}_k^e &= \underline{\hat{x}}_k^p + \mathbf{K}_k((\underline{\hat{y}}_k - \hat{\delta}_k) - \underline{\hat{x}}_k^p) \quad \text{and} \\ \mathbf{C}_k^e &= (\mathbf{I} - \mathbf{K}_k)\mathbf{C}_k^p \ ,\end{aligned}$$

with Kalman gain $\mathbf{K}_k = \mathbf{C}_k^p(\mathbf{C}_k^p + \mathbf{C}_k^\delta)^{-1}$.

B. Prediction Step

Since $\underline{\boldsymbol{x}}_k$ is the result of a nonlinear transformation of the original target positions, the system equation for the temporal evolution of $\underline{\boldsymbol{x}}_k$ is nonlinear as well. The system equation of the transformed problem can be written in the form (see Fig. 1)

$$\underline{\boldsymbol{x}}_{k+1} = a(\underline{\boldsymbol{x}}_k, \underline{\boldsymbol{w}}_k, \underline{\hat{d}}_k) = \tag{5}$$
$$\begin{bmatrix} \min\left\{\boldsymbol{l}_k^x + \boldsymbol{w}_{1,k}^x, \boldsymbol{u}_k^x + \boldsymbol{w}_{2,k}^x\right\} \\ \max\left\{\boldsymbol{l}_k^x + \boldsymbol{w}_{1,k}^x, \boldsymbol{u}_k^x + \boldsymbol{w}_{2,k}^x\right\} \\ \min\left\{\boldsymbol{l}_k^y + \boldsymbol{w}_{1,k}^y, \boldsymbol{u}_k^y + \boldsymbol{w}_{2,k}^y\right\} \\ \max\left\{\boldsymbol{l}_k^y + \boldsymbol{w}_{1,k}^y, \boldsymbol{u}_k^y + \boldsymbol{w}_{2,k}^y\right\} \\ \min\left\{\boldsymbol{l}_k^s + \boldsymbol{w}_{1,k}^s, \boldsymbol{u}_k^s + \boldsymbol{w}_{2,k}^s\right\} \\ \max\left\{\boldsymbol{l}_k^s + \boldsymbol{w}_{1,k}^s, \boldsymbol{u}_k^s + \boldsymbol{w}_{2,k}^s\right\} \end{bmatrix} + \begin{bmatrix} \hat{d}_k^x \\ \hat{d}_k^x \\ \hat{d}_k^y \\ \hat{d}_k^y \\ \hat{d}_k^x + \hat{d}_k^y \\ \hat{d}_k^x + \hat{d}_k^y \end{bmatrix} ,$$

where either $\boldsymbol{w}_{1,k}^s = \boldsymbol{w}_{1,k}^x + \boldsymbol{w}_{1,k}^y$ and $\boldsymbol{w}_{2,k}^s = \boldsymbol{w}_{2,k}^x + \boldsymbol{w}_{2,k}^y$ or $\boldsymbol{w}_{1,k}^s = \boldsymbol{w}_{2,k}^x + \boldsymbol{w}_{2,k}^y$ and $\boldsymbol{w}_{2,k}^s = \boldsymbol{w}_{1,k}^x + \boldsymbol{w}_{1,k}^y$. In order to avoid a case distinction for the noise terms $\boldsymbol{w}_{1,k}^s$ and $\boldsymbol{w}_{2,k}^s$ in (5), the noise terms can be assumed to be uncorrelated to $\boldsymbol{w}_{2,k}^x + \boldsymbol{w}_{2,k}^y$ or $\boldsymbol{w}_{1,k}^x + \boldsymbol{w}_{2,1}^y$. Hence, $\boldsymbol{w}_{1,k}^s$ and $\boldsymbol{w}_{2,k}^s$ are modeled as Gaussian noise with variance $c_{w,k}^x + c_{w,k}^y$ that is uncorrelated to $\boldsymbol{w}_k^x$ and $\boldsymbol{w}_k^y$. The prediction step can be performed with a standard nonlinear state estimator like the UKF.

C. Point Estimates

Typically one is interested in a point estimate of the target positions. In order to obtain such a point estimate from the current estimate $p(\underline{x}_k|\underline{\hat{y}}_{1:k}) = \mathcal{N}(\underline{\hat{x}}_k^e, \mathbf{C}_k^e)$, one could perform a stochastic forward mapping in order to compute probabilities densities for the target positions $\underline{z}_{1,k}$ and $\underline{z}_{2,k}$. Then, a point estimate is given by the mean of these densities.

A more simple but less accurate solution is to directly take the mean $\underline{\hat{x}}_k^e$ and set the point estimate to

$$\left[l_k^x, l_k^y\right]^T \text{ and } \left[u_k^x, u_k^y\right]^T \ ,$$

if $|\min\{l_k^x + l_k^y, u_k^x + u_k^y\} - l_k^s| < |\min\{l_k^x + u_k^y, u_k^x + l_k^y\} - l_k^s|$ and otherwise

$$\left[l_k^x, u_k^y\right]^T \text{ and } \left[u_k^x, l_k^y\right]^T \ .$$

In the same manner, one could also compute the uncertainty of the target positions.

V. Relationship to Other Approaches

The concept of transforming the original state of a system into a higher dimensional state has also been used in the context of nonlinear state estimation [25], [26], for instance. Actually, the *Unique State filter* can be seen as a non-minimal state filter [25] for the *SME filter* [17], [18], [19]. In general, each symmetric transformation for the *SME filter* can also be used to construct a *Unique State filter*. However, it must be ensured that the prediction and measurement update can be performed in the transformed state space. Furthermore, it should be possible to efficiently reconstruct the Cartesian coordinates of the targets based on the transformed state.

VI. Evaluation

In the following, we demonstrate the performance of the *Unique State filter*. For this purpose, we consider two example scenarios, whereas the *Unique State filter* is compared with the *SME filter*. The used symmetric measurement equation is given by the *sum-of-powers* equation [18], [19]

$$T(\underline{x}_k) := \begin{bmatrix} x_{1,k} + x_{2,k} \\ y_{1,k} + y_{2,k} \\ (x_{1,k})^2 + (x_{2,k})^2 - (y_{1,k})^2 - (y_{2,k})^2 \\ 2x_{1,k}y_{1,k} + 2x_{2,k}y_{2,k} \end{bmatrix} \ . \tag{6}$$

The solution of the *SME filter* is not unique, i.e., two possible solutions are $\left[\underline{z}_{1,k}, \underline{z}_{2,k}\right]^T$ and $\left[\underline{z}_{2,k}, \underline{z}_{1,k}\right]^T$ due to the symmetric measurement equation. As a consequence, the likelihood function is multimodal. Hence, a Gaussian distribution is not suitable for capturing the uncertainty about the state (in the case of closely spaced targets). The following two examples provide detailed simulation results.

A. Two Static Targets

In the first scenario, two targets are located at fixed positions $\underline{z}_{1,k} = [3,2]^T$ and $\underline{z}_{1,k} = [3.5,3]^T$, i.e., they do not move. The measurement noise is $\mathrm{diag}([1,1])$, which is quite high in comparison to the distance of the two targets. A priori it is known that one target is located at the position $[3,1]^T$ and one at the position $[3.5,1.5]^T$, both with covariance matrix $\mathrm{diag}([1,1])$. We tested the *SME filter* with the symmetric measurement equation (6) and the new *Unique State filter* both with an UKF implementation.

The resulting error for the first 200 time steps averaged over 100 runs is depicted in Fig. 3. The error [10] between true target set $Z = \{\underline{z}_1, \underline{z}_2\}$ and the estimated target set $Y = \{\underline{y}_1, \underline{y}_2\}$ is computed by means of the formula

$$d(X,Y) = (\tfrac{1}{2}\min_{\pi \in \Pi_2} \sum_{i=1,2} ||\underline{y}_i - \underline{z}_{\pi(i)}||^2)^{\frac{1}{2}} \ ,$$

where Π_2 is the set of all permutations on $\{1, 2\}$. In Fig. 3, it can be seen that the *Unique State filter* outperforms the *SME filter*. Especially, at the first 100 time steps, the estimates of the *SME filter* jump around the two true positions, which is caused by the multimodal likelihood resulting from the symmetric measurement equation. The *Unique State filter* does not suffer from this problem due to its unique state representation.

B. Two Crossing Targets

The next scenario considers two collectively moving targets, whose tracks are crossing. This is an important scenario for real-world applications and one of the most challenging tasks in target tracking. Therein, the difficulty is that the two tracks cannot be distinguished due to the kinematics of the targets, since the motion models are the same.

The starting positions of the two targets are $\underline{z}_{1,k} = [1.5, 1]^T$ and $\underline{z}_{1,k} = [2, 4]^T$. The two targets approach up to a distance of approximately 1, then their paths cross. The true (unknown) path taken by the two targets is depicted in Fig. 3a. The temporal evolution of the target position is modeled with the linear system model (2) with system noise with covariance $\text{diag}([0.002, 0.002])$ and system input $[2, 0]^T$. A priori it is known that one target is located at the position $[1.2, 2.5]^T$ and one at the position $[2.2, 0.8]^T$, both with covariance matrix $\text{diag}([0.5, 0.5])$. Again, the measurement noise is $\text{diag}([1, 1])$. The average error for 50 runs is shown in Fig. 3b. There, it can be seen that the new method provides more accurate estimation results than the *SME filter*, especially when the two targets are crossing.

VII. Conclusions and Future Work

In this paper, we introduced the so-called *Unique State filter*, which allows for association-free tracking of two targets. The basic idea is that the association problem disappears, when a representation of the two targets that eliminates the target identity is chosen. To this end, we represent the Cartesian coordinates of the two targets by means of its extreme values on the axes, and the Manhattan distance from the origin. For this state representation, it is possible to derive a measurement equation, which does not require any kind of data association. Furthermore, the resulting likelihood function is unimodal, which renders the proposed method suitable for closely spaced targets. The performance of the *Unique State filter* has been demonstrated by means of simulations.

Future work will be concerned with extending the *Unique State filter* to higher dimensions and larger target numbers. In this context, proper state representations for the *Unique State filter* have to be investigated and evaluated. For instance, techniques from order statistics [27] could be applied for this purpose. Future work also consists of extending the *Unique State filter* for dealing with false measurements and detection probabilities. Furthermore, it would be interesting to construct a symmetric measurement equation [17], [18], [19] by means of extreme values or order statistics.

References

[1] D. Hall and J. Llinas, *Handbook of Multisensor Data Fusion*. CRC Press, May 2001.

[2] M. Baum and U. D. Hanebeck, "Random Hypersurface Models for Extended Object Tracking," in *Proceedings of the 9th IEEE International Symposium on Signal Processing and Information Technology (ISSPIT 2009)*, Ajman, United Arab Emirates, Dec. 2009.

[3] J. Vermaak, N. Ikoma, and S. Godsill, "Extended object tracking using particle techniques," *IEEE Aerospace Conference*, vol. 3, pp. –1885 Vol.3, March 2004.

[4] D. Salmond and N. Gordon, "Group and Extended Object Tracking," *IEEE Colloquium on Target Tracking: Algorithms and Applications*, pp. 16/1–16/4, 1999.

[5] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman filter: Particle filters for tracking applications*. Artech House Publishers, 2004.

[6] M. J. Waxman and O. E. Drummond, "A Bibliography of Cluster (Group) Tracking," *Signal and Data Processing of Small Targets*, vol. 5428, no. 1, pp. 551–560, 2004.

[7] N. Ikoma and S. Godsill, "Extended Object Tracking with Unknown Association, Missing Observations, and Clutter Using Particle Filters," in *IEEE Workshop on Statistical Signal Processing*, sept.-1 oct. 2003, pp. 502 – 505.

[8] X. Rong Li and Y. Bar-Shalom, "Tracking in Clutter With Nearest Neighbor Filters: Analysis and Performance," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 3, pp. 995 –1010, July 1996.

[9] Y. Bar-Shalom, F. Daum, and J. Huang, "The Probabilistic Data Association Filter," *IEEE Control Systems Magazine*, vol. 29, no. 6, pp. 82 –100, Decembre 2009.

[10] L. Svensson, D. Svensson, and P. Willett, "Set JPDA Algorithm for Tracking Unordered Sets of Targets," in *12th International Conference on Information Fusion (FUSION 2009)*, 6-9 2009, pp. 1187 –1194.

[11] J. Vermaak, S. Godsill, and P. Perez, "Monte Carlo Filtering for Multi Target Tracking and Data Association," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 1, pp. 309 – 332, January 2005.

[12] H. Blom and E. Bloem, "Permutation Invariance in Bayesian Estimation of Two Targets That Maneuver In and Out Formation Flight," in *12th International Conference on Information Fusion (FUSION '09)*, 6-9 2009, pp. 1296 –1303.

[13] S. Blackman, "Multiple Hypothesis Tracking for Multiple Target Trackings," *IEEE Aerospace and Electronic Systems Magazine*, vol. 19, no. 1, pp. 5 –18, January 2004.

[14] D. Crouse, M. Guerriero, P. Willett, R. Streit, and D. Dunham, "A Look at the PMHT," in *12th International Conference on Information Fusion (FUSION '09)*, July 2009, pp. 332 –339.

[15] R. Mahler, ""Statistics 101" For Multisensor, Multitarget Data fusion," *IEEE Aerospace and Electronic Systems Magazine*, vol. 19, no. 1, pp. 53 –64, January 2004.

[16] ——, "Multitarget Bayes Filtering via First-Order Multitarget Moments," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 39, no. 4, pp. 1152 – 1178, Octobre 2003.

[17] E. Kamen, "Multiple Target Tracking based on Symmetric Measurement Equations," *IEEE Transactions on Automatic Control*, vol. 37, no. 3, pp. 371 –374, March 1992.

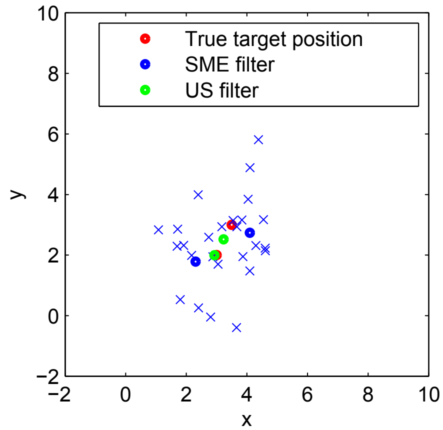
[18] W. Leven and A. Lanterman, "Unscented Kalman Filters for Multiple Target Tracking With Symmetric Measurement Equations," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 370 –375, February 2009.

[19] ——, "Multiple Target Tracking with Symmetric Measurement Equations using Unscented Kalman and Particle Filters," in *Proceedings of the Thirty-Sixth Southeastern Symposium on System Theory*, 2004, pp. 195 – 199.

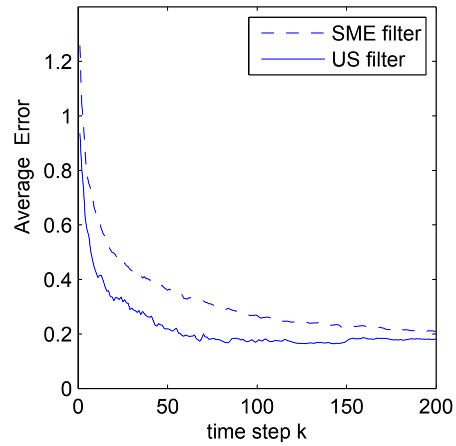
[20] D. Simon, *Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches*, 1st ed. Wiley & Sons, August 2006.

[21] A. Krause, J. Leskovec, and C. Guestrin, "Data Association For Topic Intensity Tracking," in *Proceedings of the 23rd International Conference on Machine Learning (ICML 2006)*. New York, NY, USA: ACM, 2006, pp. 497–504.

[22] S. Nadarajah and S. Kotz, "Exact Distribution of the Max/Min of Two Gaussian Random Variables," *IEEE Trans. Very Large Scale Integr. Syst.*, vol. 16, no. 2, pp. 210–212, 2008.

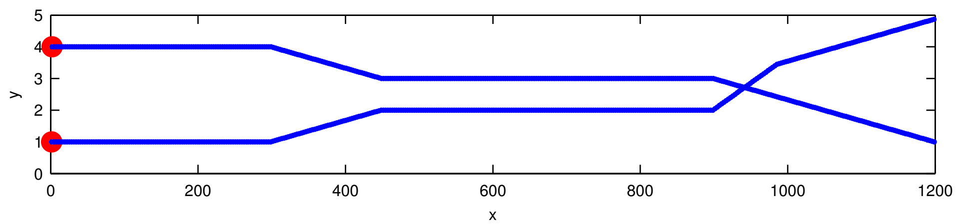


(a) Example run: All received measurements are marked by crosses.

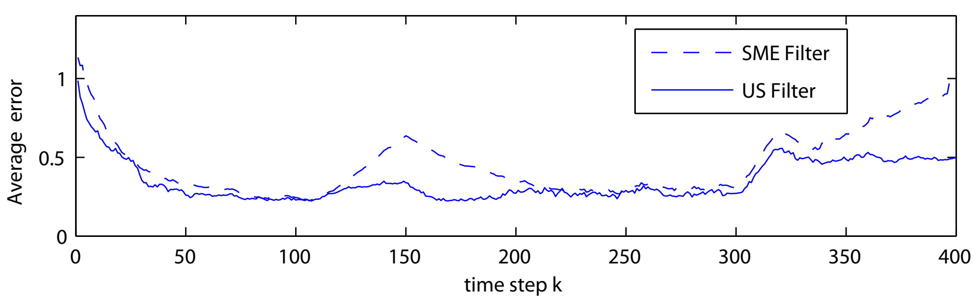


(b) Average error.

Fig. 2: Simulation results for the scenario with static targets. A screenshot of an example run (a) and the average error (b) for the first 200 time steps averaged over 100 runs are depicted.



(a) Path taken by the two targets.



(b) Average error for 400 time step.

Fig. 3: Simulation results for two collectively moving targets with crossing tracks. The path of the two targets (a) and the average error (b) for the first 400 time steps averaged over 50 runs are shown.

[23] S. J. Julier and J. K. Uhlmann, "Unscented Filtering and Nonlinear Estimation," in *Proceedings of the IEEE*, vol. 92, no. 3, 2004, pp. 401–422.

[24] M. F. Huber and U. D. Hanebeck, "Gaussian Filter based on Deterministic Sampling for High Quality Nonlinear Estimation," in *Proceedings of the 17th IFAC World Congress (IFAC 2008)*, Seoul, Korea, July 2008.

[25] T. Lefebvre, H. Bruyninckx, and J. De Schutter, *Nonlinear Kalman Filtering for Force-Controlled Robot Tasks*. Springer Berlin / Heidelberg, 2005, ch. The Non-Minimal State Kalman Filter, pp. 77–94.

[26] U. D. Hanebeck, "Optimal Filtering of Nonlinear Systems Based on Pseudo Gaussian Densities," in *Proceedings of the 13th IFAC Symposium on System Identification (SYSID 2003)*, Rotterdam, Netherlands, Aug. 2003, pp. 331–336.

[27] H. David and H. Nagaraja, *Order statistics*. John Wiley & Sons, 2003.