

## Certiably Globally Optimal Unsupervised Machine Learning

It is known [Jean B. Lasserre, 2001] that for polynomial problems (POPs), i.e., nonconvex optimization problems where objective and constraints are multivariate polynomials, the global optimum can be determined via rank relaxation, adding redundant constraints, ascending Lasserre's hierarchy, and solving a semidefinite program (SDP), in polynomial time. While this methodology is not yet real-time applicable as of 2024, it is merely a matter of time until increased computational power, more advanced solvers, and optimized problem design strategies render this possible. The potential applications of machine learning with optimality guarantees and certificates are immense. Many supervised machine learning and perception problems have already been formulated as POP and successfully solved with this framework, e.g., point cloud registration, pose and shape estimation, and robust estimation.

Unsupervised ML problems, like clustering and Gaussian mixture estimation, can also be stated as POP. The goal of this work is demonstrating and evaluating globally and certiably solving these kind of problems with that same framework as well. The state of art closest to this in terms of problem structure is robust perception, which should be reproduced first to get familiar with the required methodology and tools.

### What to do

- Get familiar with solving POPs via SDP  
[Dümbgen: Toward Globally Optimal State Estimation Using Automatically Tightened Semidefinite Relaxations]
- Reproduce robust perception  
[Yang: Certifiably Optimal Outlier-Robust Geometric Perception: Semidefinite Relaxations and Scalable Global Optimization]
- Attempt Clustering
- Attempt Gaussian Mixture Estimation
- Optimize problem structure, e.g. exploiting sparsity
- Evaluate against state of art

### Requirements:

Students with a background in computer science, mathematics, physics, electrical engineering, or other engineering majors. Excellent grades are required, please send "Certificate of all Courses and Grades". Pre-knowledge in Julia, Matlab, or Python are welcome. Strong self-motivation, perseverance, reliability, mathematical skills, and critical mind are expected.

### Emphasis:

Theoretical Study

Software Implementation

Hardware Implementation

### We offer:

- excellent support and advice
- highend infrastructure
- contact to industry and research partners

### Contact:

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