The Kernel-SME Filter with Adaptive Kernel Widths for Association-free Multi-target Tracking

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Abstract— Different objectives and paradigms exist for tracking multiple targets when measurements do not contain information about the target identities (IDs). The Symmetric Measurement Equation (SME) filter can be used when one is agnostic to the labels and does not attempt to assign different IDs to the different targets. We present an extension of the Kernel-SME filter that, unlike the original variant, uses adaptive kernel widths that depend on the respective uncertainty. In our evaluation, it outperformed existing SME-based approaches, while it is only second to a more complex global nearest neighbor tracker.

I. Introduction

Multi-target Tracking (MTT) [1] is a widespread problem in a variety of application domains that is nontrivial. One of the major challenges in MTT is the unknown measurement-to-target associations, which is also referred to as the data association problem. Several of the existing MTT methods solve the data association problem based on an explicit enumeration of association hypotheses.

If multiple association hypotheses are considered, as in Multi-Hypothesis Tracking (MHT) [2], the number of possible combinations in explicit enumeration grows exponentially with the horizon over which the elements are maintained. Consequently, the computational complexity increases for a larger number of targets, and hence, the practical application of these methods requires fast approximations.

In this paper, we consider association-free MTT methods based on a permutation-invariant measurement equation. The focus of this paper lies on methods that are based on Symmetric Measurement Equations (SME) [3], [4]. SME-based methods use symmetric transformations to modify the measurement equation, leading to permutation-invariant pseudo-measurements. Due to the permutation invariance, no association hypothesis is required.

One of the simplest approaches for the data association problem is based on local nearest neighbor (LNN) associations [5], which assign each target the closest measurement with respect to a distance metric. The global nearest neighbor (GNN) association [5] seeks to find the association hypothesis that minimizes the sum of distances between measurements and targets, which is equivalent to the maximum a posteriori (MAP) hypothesis. The MHT [2] approaches the problem by maintaining the track history in a tree-like structure, which allows the correction of a wrong assignment at a later time step. Hence, the MHT can be seen as an extension of the GNN that estimates the MAP association hypothesis over multiple time steps. However, maintaining and managing the hypotheses over time is highly computationally expensive, which is why state-of-the-art methods commonly use techniques to reduce the complexity. The algorithms mentioned so far are based on hard association decisions, i.e., exactly one measurement is allocated to each target (though the decision is delayed in the MHT).

Contrary to these methods, the Joint Integrated Probabilistic Data Association Filter (JPDA) [6], [7] considers a probabilistic approach for the data association problem where the target state estimate is based on multiple measurements weighted by their marginal association probabilities. The estimate provided by the JPDAF is the minimum mean squared error (MMSE) estimate for the respective time step (it is generally not optimal when used consecutively over multiple time steps). Furthermore, as the computational complexity to determine the marginal association probabilities increases exponentially with the number of targets, fast approximations are typically used in practical applications [8].

Contrary to the previously mentioned methods, there are association-free approaches that avoid the data association problem and the involved combinatorial complexity. The Probability Hypothesis Density (PHD) filter [9] maintains the first-order moment of the random finite set and thus avoids explicit enumeration. However, by maintaining the first-order moment only, other important information about the targets is lost. Another approach is based on an SME [3], [4], which modifies the measurement equation by applying a symmetric transformation. The modified measurement equation is permutation-invariant with respect to the order of the measurements, and thus no association hypothesis is required for the measurement update of the multi-target state estimate.

Association-free Direct Filtering (DF) [10], [11] represents the measurement as an unordered set without any association. The measurement update is performed based on the gradients of a point set distance measure.

The Kernel-SME [12], [13] filter generalizes the SME

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approach by using kernel transformations to represent the measurements as Gaussian mixtures. For the measurement update, the Gaussian mixture has to be discretized at specific locations. Furthermore, the Gaussian kernels used in the kernel transformation depend on the kernel width that needs to be selected appropriately and thus impose a parameter selection problem.

In this paper, we focus on the comparison of different SME-based MTT algorithms. Therefore, we introduce a general methodology for association-free MTT. We specifically consider simulations with difficult scenarios, such as closely spaced and crossing target trajectories. Furthermore, we introduce an adaptive parameter selection method for the Kernel-SME filter that integrates the uncertainty of the target state estimate in each time step. We provide a comprehensive comparison of the considered association-free MTT algorithms with other existing methods.

The remainder of this paper is structured as follows: In the next section, we give a description of the MTT problem and the underlying association uncertainty. In Sec. III, we introduce the general methodology of association-free direct filtering based on SMEs and describe the algorithms based on this concept. Sec. IV introduces a simulation that we use to compare the results of the algorithms with other methods. Finally, we give a conclusion about the considered methods in Sec. V.

II. Problem Formulation

We consider tracking N moving targets with locations $\underline{\boldsymbol{x}}_k^i \in \mathbb{R}^n$ according to some distribution (bold font letters are used for random variables) with indices $i = 1, \dots, N$. The multi-target state can be described as a random vector $\underline{\boldsymbol{x}}_k := \left[(\underline{\boldsymbol{x}}_k^1)^\top, \dots, (\underline{\boldsymbol{x}}_k^N)^\top\right]^\top$ or (if the targets are not to be distinguished over time) an unordered set $\boldsymbol{\mathcal{X}} := \{\underline{\boldsymbol{x}}_k^1, \dots, \underline{\boldsymbol{x}}_k^N\}$.

The evolution of each state is described by the linear system model

$$\underline{\boldsymbol{x}}_{k+1}^i = \mathbf{A}_k^i \underline{\boldsymbol{x}}_k^i + \underline{\boldsymbol{w}}_k^i \ ,$$

where $\underline{\boldsymbol{w}}_k^i$ is zero-mean white noise with covariance $\boldsymbol{\Sigma}_k^w$. Each target has exactly one associated measurement $\underline{\boldsymbol{y}}_k^i \in \mathbb{R}^n$ such that in each time step k, there are N measurements, which are described using an unordered set $\boldsymbol{\mathcal{Y}}_k := \{\underline{\boldsymbol{y}}_k^1, \dots, \underline{\boldsymbol{y}}_k^N\}$. Each measurement has an associated measurement equation

$$\underline{\boldsymbol{y}}_k^{\pi_k(i)} = \mathbf{H}_k^i \underline{\boldsymbol{x}}_k^i + \underline{\boldsymbol{v}}_k^i \ ,$$

where $\underline{\boldsymbol{v}}_k^i$ describes a zero-mean additive noise with covariance Σ_k^v and π_k is the permutation that specifies the measurement-to-track associations. Given the permutation π_k, the measurements can be represented as a stacked multi-target measurement vector

$$P_{\pi_k}(\underline{\boldsymbol{y}}_k) = \left[\left(\underline{\boldsymbol{y}}_k^{\pi_k(1)}\right)^\top, \ \dots, \ \left(\underline{\boldsymbol{y}}_k^{\pi_k(N)}\right)^\top\right]^\top \ ,$$

where $\underline{\boldsymbol{y}}_k = \left[(\underline{\boldsymbol{y}}_k^1)^\top, \dots, (\underline{\boldsymbol{y}}_k^N)^\top\right]^\top$, such that the multi-target measurement equation can be described by

$$P_{\pi_k}(\underline{\boldsymbol{y}}_k) = \begin{bmatrix} \mathbf{H}_k^1 & & \\ & \ddots & \\ & & \mathbf{H}_k^N \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{x}}_k^1 \\ \vdots \\ \underline{\boldsymbol{x}}_k^N \end{bmatrix} + \begin{bmatrix} \underline{\boldsymbol{v}}_k^1 \\ \vdots \\ \underline{\boldsymbol{v}}_k^N \end{bmatrix} .$$

However, the representation of the measurements as a stacked vector implies a specific order, which is described by the permutation π_k. To apply the measurement update, most methods rely on solving the data association problem to obtain the measurement-to-track associations. Based on the association hypothesis, standard filtering methods can be applied to determine the posterior state estimate.

Solving the data association problem explicitly allows generating entire trajectories for the individual targets. However, in some applications one tracks objects without consideration of their identity. In this case, avoiding data association may lead to more robust or significantly faster results. Such approaches are referred to as association-free trackers. One such approach is the Kernel-SME filter [12], which is the approach we improve in this paper.

III. Association-Free Multi-target Tracking

In the following, we will give a more detailed description of Bayesian filtering using pseudo-measurements. We begin with a general introduction to the underlying concepts. Furthermore, we introduce three methods based on the SME approach.

A. Association-Free Measurement Equation

The key idea of an association-free measurement equation is to modify the original measurement equation in such a way that it becomes invariant with respect to the permutation π_k. Therefore, we consider a function $\underline{S}$ that can be applied to the measurement equation

$$\underline{\boldsymbol{s}} = \underline{S}\left(\boldsymbol{\mathcal{Y}}, \underline{\boldsymbol{x}}\right),$$

resulting in permutation-invariant pseudo-measurements $\underline{\boldsymbol{s}}$. Based on these pseudo-measurements, consider a Gaussian approximation of the posterior density, i.e.,

$$p(\underline{x} \,|\, \underline{\boldsymbol{s}}) = \mathcal{N}\left(\underline{x}\,;\, \underline{\mu}_{k|k}^x, \boldsymbol{\Sigma}_{k|k}^x\right) \ ,$$

where $\underline{\mu}_{k|k}^x$ is the posterior state estimate and $\Sigma_{k|k}^x$ is the posterior covariance matrix. The target state can be estimated using the Linear Minimum Mean Squared Error estimator [5]

$$\begin{aligned} \underline{\mu}_{k|k}^x &= \underline{\mu}_{k|k-1}^x + \boldsymbol{\Sigma}_k^{xs} \left(\boldsymbol{\Sigma}_k^{ss}\right)^{-1} \boldsymbol{\Sigma}_k^{sx} \left(\underline{\boldsymbol{s}}_k - \underline{\mu}_k^s\right) \ , \\ \boldsymbol{\Sigma}_{k|k}^x &= \boldsymbol{\Sigma}_{k|k-1}^x + \boldsymbol{\Sigma}_k^{xs} \left(\boldsymbol{\Sigma}_k^{ss}\right)^{-1} \boldsymbol{\Sigma}_k^{sx} \ , \end{aligned} \tag{1}$$

where $\underline{\mu}_k^s$ is the pseudo-measurement mean, $\boldsymbol{\Sigma}_k^{ss}$ the covariance matrix, and $\boldsymbol{\Sigma}_k^{sx}$ the cross-covariance matrix of the pseudo-measurement and prior state estimate.

B. Symmetric Measurement Equation

SMEs were first introduced by Kamen [3], [4]. They are based on the idea of modifying the measurement equation by applying a symmetric transformation $\underline{S} : \mathbb{R}^{N \cdot n} \to \mathbb{R}^{N_a}$. This results in

$$\underline{\boldsymbol{s}}_k := \underline{S}(\underline{\boldsymbol{y}}_k) = \underline{S}(P_{\pi_k}(\underline{\boldsymbol{y}}_k)) \ , \ \forall \pi_k \in \Pi_N \ ,$$

with Π_N being the set of all possible permutations from 1 to N. Hence, the data association problem can be bypassed by replacing the original measurement vector $\underline{\boldsymbol{y}}_k$ with the pseudo-measurement vector $\underline{\boldsymbol{s}}_k$ in the measurement update.

To give some examples of symmetric transformations used for the MTT, we consider the sum of power function [14], [15]

$$\underline{\boldsymbol{s}}_k^{\mathrm{pow}} = \underline{S}^{\mathrm{pow}}([y_k^1, y_k^2]) = \begin{bmatrix} y_k^1 + y_k^2 \\ (y_k^1)^2 + (y_k^2)^2 \end{bmatrix},$$

and the sum of products function

$$\underline{\boldsymbol{s}}_k^{\mathrm{prod}} = \underline{S}^{\mathrm{prod}}([y_k^1, y_k^2, y_k^3]) = \begin{bmatrix} y_k^1 + y_k^2 + y_k^3 \\ y_k^1 \cdot y_k^2 + y_k^1 \cdot y_k^3 + y_k^2 \cdot y_k^3 \\ y_k^1 \cdot y_k^2 \cdot y_k^3 \end{bmatrix}.$$

By just looking at both functions, it is evident that the value of the pseudo-measurements is not affected by the order of the measurements. However, the avoidance of the data association problem comes with other difficulties. First, selecting a transformation $\underline{S}$ such that the information contained in the measurements is encapsulated in the pseudo-measurements can be challenging. Second, the transformed symmetric measurement equation is highly nonlinear, and thus requires nonlinear filtering techniques, such as the extended Kalman filter [16] or unscented Kalman filter (UKF) [17], for the measurement update.

C. Association-Free Direct Filtering

The key idea of association-free direct filtering [10], [11] is to employ a point set distance measure $D(\cdot, \cdot)$ that quantifies the distance between two sets of particles. Considering two sets $\boldsymbol{\mathcal{Y}}$ and $\boldsymbol{\mathcal{X}}$, the distance between both sets can be determined by

$$d = D(\boldsymbol{\mathcal{Y}}, \boldsymbol{\mathcal{X}}) = D\left(\{\underline{\boldsymbol{y}}^1, \ldots, \underline{\boldsymbol{y}}^{N_y}\}, \{\underline{\boldsymbol{x}}^1, \ldots, \underline{\boldsymbol{x}}^{N_x}\}\right) \ .$$

We now demonstrate how to derive pseudo-measurements using a point set distance measure D. Therefore, we assume D to be differentiable with respect to $\underline{\boldsymbol{x}}_k^i$. Under this assumption, pseudo-measurements can be obtained by considering the gradients

$$\underline{\boldsymbol{s}}_k = \frac{\partial D(\boldsymbol{\mathcal{Y}}, \boldsymbol{\mathcal{X}})}{\partial \underline{\boldsymbol{x}}_k} = \underline{G}(\boldsymbol{\mathcal{Y}}, \underline{\boldsymbol{x}}_k) \ .$$

This method provides a flexible way to derive pseudo-measurements, which only requires the point set distance to be differentiable. In this paper, we consider the Localized Cumulative Distribution [18], [19], which is the basis for the generalized Cramér–von Mises distance. For the measurement update, the random variable $\underline{\boldsymbol{x}}_k^{\mathrm{p}}$ is assumed to be Gaussian distributed with mean $\underline{\mu}_{k|k-1}^x$ and covariance $\Sigma_{k|k-1}^x$. Under this assumption, we can generate P samples $(\underline{\boldsymbol{x}}_k^{\mathrm{p}})^{(i)}$ of $\underline{\boldsymbol{x}}_k^{\mathrm{p}}$ and similarly P random finite sets $\{\boldsymbol{\mathcal{Y}}^{(1)}, \ldots, \boldsymbol{\mathcal{Y}}^{(P)}\}$ for the measurement set $\boldsymbol{\mathcal{Y}}$. Since the pseudo-measurements are permutation-invariant, the P measurement sets can be represented as stacked vectors $\underline{\boldsymbol{y}}_k^{(i)}$. Thus, we can define the stacked random variable

$$\underline{\boldsymbol{z}}_k = \begin{bmatrix} \underline{\boldsymbol{x}}_k^{\mathrm{p}} \\ \underline{\boldsymbol{y}}_k \end{bmatrix} \in \mathbb{R}^{2 \cdot N \cdot n} \ ,$$

which is Gaussian distributed with mean

$$\hat{\underline{z}}_k := \begin{bmatrix} \underline{\mu}_{k|k-1}^x \\ \hat{\underline{y}}_k \end{bmatrix},$$

and covariance matrix

$$\boldsymbol{\Sigma}_k^z := \begin{bmatrix} \boldsymbol{\Sigma}_{k|k-1}^x & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_k^y \end{bmatrix}.$$

Considering P standard Gaussian distributed samples $\underline{\boldsymbol{g}}^{(i)} \in \mathbb{R}^{2 \cdot N \cdot n}$, we define

$$\mathbf{G} := \left[\underline{\boldsymbol{g}}^{(1)}, \ldots, \underline{\boldsymbol{g}}^{(P)}\right] \in \mathbb{R}^{(2 \cdot N \cdot n) \times P} \ .$$

To obtain samples that represent a standard Gaussian distribution most effectively, we consider deterministic samples based on the Smart Sampling Kalman Filter [20], [21]. These samples are generated based on a low-discrepancy Dirac Mixture approximation of the standard Gaussian density with an arbitrarily-chosen number of samples. To obtain samples with mean $\hat{\underline{z}}_k$ and covariance $\boldsymbol{\Sigma}_k^z$, we use the Cholesky decomposition for the covariance $\boldsymbol{\Sigma}_k^z = \boldsymbol{\mathcal{C}}^z (\boldsymbol{\mathcal{C}}^z)^\top$ to transform the standard Gaussian samples according to

$$\mathbf{Z} = \boldsymbol{\mathcal{C}}^z \cdot \mathbf{G} + \hat{\underline{z}}_k \cdot (\underline{1}_{\mathrm{p}})^\top,$$

where $\underline{1}_{\mathrm{p}} \in \mathbb{R}^{\mathrm{p}}$ is a vector containing all ones. The columns of the matrix $\mathbf{Z} = [\underline{\boldsymbol{z}}_k^{(1)}, \ldots, \underline{\boldsymbol{z}}_k^{(P)}]$ represent the generated samples of $\underline{\boldsymbol{z}}_k$ with mean $\hat{\underline{z}}_k$ and covariance $\boldsymbol{\Sigma}_k^z$. Each sample $\underline{\boldsymbol{z}}_k^{(i)}$ is used to obtain the pseudo-measurements

$$\underline{\boldsymbol{s}}_k^{(i)} = \underline{G}(\boldsymbol{\mathcal{Y}}^{(i)}, (\underline{\boldsymbol{x}}_k^{\mathrm{p}})^{(i)}),$$

which can be stacked into a sample of the pseudo-measurement

$$\mathbf{S} = \left[\underline{\boldsymbol{s}}_k^{(1)}, \ldots, \underline{\boldsymbol{s}}_k^{(P)}\right].$$

We assume that each tuple of generated samples $(\underline{\boldsymbol{x}}_k^{\mathrm{p}})^{(i)}$ and pseudo-measurements $\underline{\boldsymbol{s}}_k^{(i)}$ is jointly Gaussian distributed, i.e.,

$$\begin{bmatrix} (\underline{\boldsymbol{x}}_k^{\mathrm{p}})^{(i)} \\ \underline{\boldsymbol{s}}_k^{(i)} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \underline{x} \\ \underline{s} \end{bmatrix}; \begin{bmatrix} \underline{\mu}_{k|k-1}^x \\ \underline{\mu}_k^s \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{k|k-1}^x & \boldsymbol{\Sigma}_k^{xs} \\ \boldsymbol{\Sigma}_k^{sx} & \boldsymbol{\Sigma}_k^s \end{bmatrix}\right).$$

The required moments for the measurement update can be obtained using the sample moments $\hat{\underline{\boldsymbol{s}}}_k = \mathrm{Mean}(\mathbf{S})$ and

$$\hat{\boldsymbol{\Sigma}}_k = \begin{bmatrix} \hat{\boldsymbol{\Sigma}}_{k|k-1}^x & \hat{\boldsymbol{\Sigma}}_k^{xs} \\ \hat{\boldsymbol{\Sigma}}_k^{sx} & \hat{\boldsymbol{\Sigma}}_k^s \end{bmatrix} = \mathrm{Cov}\left(\begin{bmatrix} \mathbf{X}^{\mathrm{p}} \\ \mathbf{S} \end{bmatrix}\right) \ ,$$

where $\mathbf{X}^{\mathrm{p}} = [(\underline{\boldsymbol{x}}_k^{\mathrm{p}})^{(1)}, \ldots, (\underline{\boldsymbol{x}}_k^{\mathrm{p}})^{(P)}]$. Given these moments, the measurement update can be performed by

$$\underline{\mu}_{k|k}^{x} = \underline{\mu}_{k|k-1}^{x} + \hat{\boldsymbol{\Sigma}}_k^{xs} \left(\hat{\boldsymbol{\Sigma}}_k^{s}\right)^{-1} (\underline{\boldsymbol{s}}_k - \hat{\underline{\boldsymbol{s}}}_k) \text{ with } \underline{\boldsymbol{s}}_k = \underline{0} \ ,$$
$$\boldsymbol{\Sigma}_{k|k}^{x} = \boldsymbol{\Sigma}_{k|k-1}^{x} - \hat{\boldsymbol{\Sigma}}_k^{xs} \left(\hat{\boldsymbol{\Sigma}}_k^{s}\right)^{-1} \hat{\boldsymbol{\Sigma}}_k^{sx} \ .$$

The measurement update is similar to the one we considered in (1), where the analytic moments are replaced with the sample moments and the pseudo-measurement is set to zero.

Furthermore, this approach can be extended by augmenting the pseudo-measurements. We can include symmetric transformations, such as the sum of powers and the sum of products we introduced for the standard SME approach. Similar to the gradients of the point set distance, we can include the pseudo-measurements $(\underline{\boldsymbol{s}}_k^{\mathrm{pow}})^{(i)}$ and $(\underline{\boldsymbol{s}}_k^{\mathrm{prod}})^{(i)}$, which can be combined with $\underline{\boldsymbol{s}}_k^{(i)}$, resulting in

$$(\underline{\boldsymbol{s}}_k^{\mathrm{comb}})^{(i)} = \begin{bmatrix} (\underline{\boldsymbol{s}}_k)^{(i)} \\ (\underline{\boldsymbol{s}}_k^{\mathrm{pow}})^{(i)} \\ (\underline{\boldsymbol{s}}_k^{\mathrm{prod}})^{(i)} \end{bmatrix} .$$

D. Kernel-SME Filter

The Kernel-SME filter [12], [13] extends the idea of the SME approach by considering kernel transformations

$$S^K(\boldsymbol{\mathcal{Y}}_k, \underline{x}) = F_{\underline{\boldsymbol{y}}_k}(\underline{x}) := \sum_{\underline{\boldsymbol{y}} \in \boldsymbol{\mathcal{Y}}_k} K(\underline{x}, \underline{\boldsymbol{y}}) \ ,$$

where K is a positive definite kernel. The kernel is per definition symmetric and due to the summation, the kernel transformation is permutation invariant. A variety of kernel transformation functions can be selected. However, one of the most commonly used kernel functions is the Gaussian kernel

$$K(\underline{x}, \underline{\boldsymbol{y}}) = \frac{1}{\sqrt{2\pi}\Gamma} \exp\left(-\frac{1}{2}\frac{\|\underline{x} - \underline{\boldsymbol{y}}\|_2^2}{\Gamma^2}\right),$$

with kernel width $\Gamma \in \mathbb{R}_+$. The Gaussian kernel has the advantage of being injective [22], and thus, it is ensured that all information of the measurements is encapsulated in the Gaussian mixture. However, applying the kernel transformation to the measurements results in a Gaussian mixture $F_{\underline{\boldsymbol{y}}_k}$, which cannot be used for the measurement update directly and is instead represented by using $N(2n+1)$ discrete test points $\underline{a}_k^1, \ldots, \underline{a}_k^{N_a}$ with

$$\underline{a}_k^{l+i-1} := \underline{\boldsymbol{y}}_k^l + \left(\sqrt{n\Gamma}\right), \quad \underline{a}_k^{l+2(i-1)} := \underline{\boldsymbol{y}}_k^l - \left(\sqrt{n\Gamma}\right) \ ,$$

for $l = 1, \ldots, n$ and $i = 1, \ldots, N$. The pseudo-measurements $\underline{\boldsymbol{s}}_k = [\underline{\boldsymbol{s}}_k^1, \ldots, \underline{\boldsymbol{s}}_k^{N_a}]$ are obtained by

$$\underline{\boldsymbol{s}}_k^i = \sum_{l=1}^{N} K(\underline{a}_k^i, \underline{\boldsymbol{y}}_k^l) \ .$$

For the measurement update, we require the moments of the pseudo-measurements, which can be determined analytically [23]. The mean of the pseudo-measurements can be determined by

$$\underline{\mu}_{k,i}^{s} = \sum_{n=1}^{N} P_l^{\boldsymbol{\Gamma}}(\underline{a}_k^n) \ ,$$

where $P_l^{\boldsymbol{\Gamma}}(\underline{a}_k^n) = \mathcal{N}\left(\underline{a}_k^n; \hat{\underline{y}}_k^l, \mathbf{H}_k^l \boldsymbol{\Sigma}_k^{\mathrm{p}} (\mathbf{H}_k^l)^\top + \boldsymbol{\Sigma}_k^v + \boldsymbol{\Gamma}\right)$, with $\boldsymbol{\Gamma} = \Gamma \cdot \mathbf{I}$. The covariance is given by

$$\boldsymbol{\Sigma}_k^{s_i s_j} = \left(\sum_{l=1}^{N} \sum_{\substack{m=1 \\ m \neq l}}^{N} \mathcal{N}\left(\begin{bmatrix} \underline{a}_k^i \\ \underline{a}_k^j \end{bmatrix}; \begin{bmatrix} \hat{\underline{y}}_k^l \\ \hat{\underline{y}}_k^m \end{bmatrix}, \boldsymbol{\Sigma}_k^{lm}\right)\right)$$
$$+\mathcal{N}\left(\underline{a}_k^i; \underline{a}_k^j, 2\boldsymbol{\Gamma}\right)\left(\sum_{l=1}^{N} P_l^{\frac{1}{2}\boldsymbol{\Gamma}}\left(\frac{\underline{a}_k^i + \underline{a}_k^j}{2}\right)\right) - \underline{\mu}_{k,i}^{s} \cdot \underline{\mu}_{k,j}^{s} \ ,$$

and the cross-covariance

$$\boldsymbol{\Sigma}^{lm} = \begin{bmatrix} \mathbf{H}_k^l & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_k^m \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma}_{k|k-1}^{x_l} & \boldsymbol{\Sigma}_{k|k-1}^{x_{lm}} \\ \boldsymbol{\Sigma}_{k|k-1}^{x_{ml}} & \boldsymbol{\Sigma}_{k|k-1}^{x_m} \end{bmatrix} \begin{bmatrix} \mathbf{H}_k^l & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_k^m \end{bmatrix}^\top + \begin{bmatrix} \boldsymbol{\Sigma}_k^{v_l} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_k^{v_m} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Gamma} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma} \end{bmatrix}.$$

Using the analytic moment calculation, the measurement update can be performed using the Kalman filter equations, as stated in (1).

E. Kernel-SME Filter with Adaptive Kernel Width

In the Kernel-SME filter, the measurements are represented as a Gaussian mixture with kernel width Γ at specific locations $\underline{a}_k^i$, where an appropriate selection of the kernel width is crucial for the accuracy of the Kernel-SME filter.

Intuitively, the kernel width should be selected such that the measurement predictions are covered by the Gaussian kernels. Therefore, it has been proposed to select the kernel width similar to the measurement noise, i.e., $\Gamma = \sigma_v^2$.

However, this policy does not consider all sources of uncertainty. Since the actual measurement only depends on the unknown true target state, deviations of the state estimates are not taken into account. A larger kernel width is desirable in situations with high uncertainty since the deviation of the true state and its estimate might be higher. On the contrary, an excessively large kernel width can lead to poor tracking performance due to oversmoothing [24]. Hence, a selection of a static kernel width might be not appropriate for scenarios with varying uncertainties.

For our approach, we consider a dynamic kernel width that integrates the uncertainty of the prior state estimate in each time step. In this paper, we consider the kernel width

$$\Gamma_k^l = \sqrt{\rho(\boldsymbol{\Sigma}_{k|k-1}^{x_l}) + \sigma_v^2} \ ,$$

where $\rho(\boldsymbol{\Sigma}_{k|k-1}^{x_l})$ is the spectral radius of the prior state covariance of $\underline{\boldsymbol{x}}_k^l$. By including the spectral radius, the adaptive kernel width integrates the uncertainty that is

described by the covariance of the prior state estimate. Consequently, the kernel width becomes larger in time steps of higher uncertainty to ensure the measurement predictions are covered by the Gaussian kernel.

IV. Application Example

We consider $N = 3$ moving targets traveling along specific trajectories in the Euclidean plane. For this, we predefined the motion of the targets in each time step. The initial true location of the targets are

$$\tilde{\underline{x}}_k^1 = \begin{bmatrix} -2.0 \\ 0.3 \end{bmatrix}, \quad \tilde{\underline{x}}_k^2 = \begin{bmatrix} -2.0 \\ 0.0 \end{bmatrix}, \quad \tilde{\underline{x}}_k^3 = \begin{bmatrix} -2.0 \\ -0.3 \end{bmatrix}.$$

The multi-target state is represented as the stacked vector $\tilde{\underline{x}}_k = [(\tilde{\underline{x}}_k^1)^\top, (\tilde{\underline{x}}_k^2)^\top, (\tilde{\underline{x}}_k^3)^\top]^\top$.

The motion of the multi-target state vector is described by the increments $\Delta\underline{x}_k$ which can be described by the model

$$\tilde{\underline{x}}_{k+1} = \tilde{\underline{x}}_k + \Delta\underline{x}_k \ .$$

The target motion increments $\Delta\underline{x}_k$ in this simulation are predefined and do not include information about the velocity. The multi-target probabilistic system model is described by

$$\underline{\boldsymbol{x}}_{k+1} = \underline{\boldsymbol{x}}_k + \Delta\underline{x}_k + \underline{\boldsymbol{w}}_k \ ,$$

where $\underline{\boldsymbol{w}}_k$ is a zero-mean Gaussian noise with covariance $\sigma_w^2\mathbf{I}_6$ and $\sigma_w = 0.03$. For the measurement model, we consider the identity model

$$\underline{\boldsymbol{y}}_k^i = \tilde{\underline{x}}_k^i + \underline{\boldsymbol{v}}_k^i \ ,$$

where $\underline{\boldsymbol{v}}_k^i$ is a zero-mean Gaussian noise with covariance $\sigma_v^2\mathbf{I}_2$ for $i = 1, 2, 3$. The measurements $\underline{\boldsymbol{y}}_k^i$ are permuted in each time step such that their order does not reveal any information about the measurement origin.

For the simulation, we focus on scenarios that are difficult for the most commonly used algorithms. To do this, we consider a scenario of closely spaced targets with crossing trajectories. For our simulation, we set the number of time steps T to 500. In each simulation run, the targets are moving in parallel for some time, followed by a phase in which the target trajectories cross. In our scenario, the target trajectories are crossing five times in total. Furthermore, we considered two scenarios with low measurement noise $\sigma^v = 0.02$ and high measurement noise $\sigma^v = 0.08$.

For comparison, we also assessed the tracking accuracy of other multi-target trackers. First, we considered assignment-based trackers in which one Kalman filter is run for each target, and the measurement-to-track associations are obtained by a GNN or LNN. For the LNN, we used the variant that allows assigning one measurement to multiple tracks. We included the JPDAF, which updates the target estimates based on the marginal association likelihoods. For the SME approach, we considered the combination of sum of power and sum products transformation where the measurement update

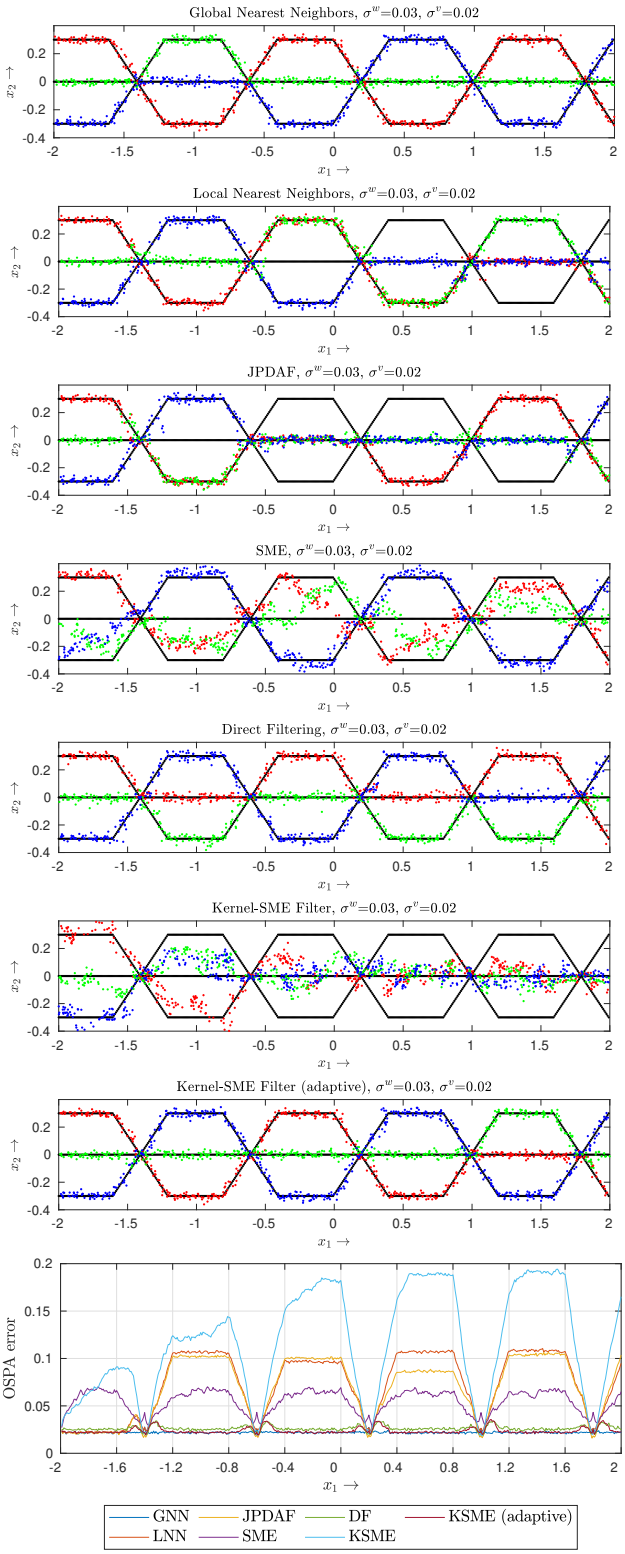


Fig. 1: Considered methods in the low noise scenario of crossing targets with five crossings. The plot at the bottom shows the average OSPA for all locations over all 100 simulation runs along the dimension x_1.

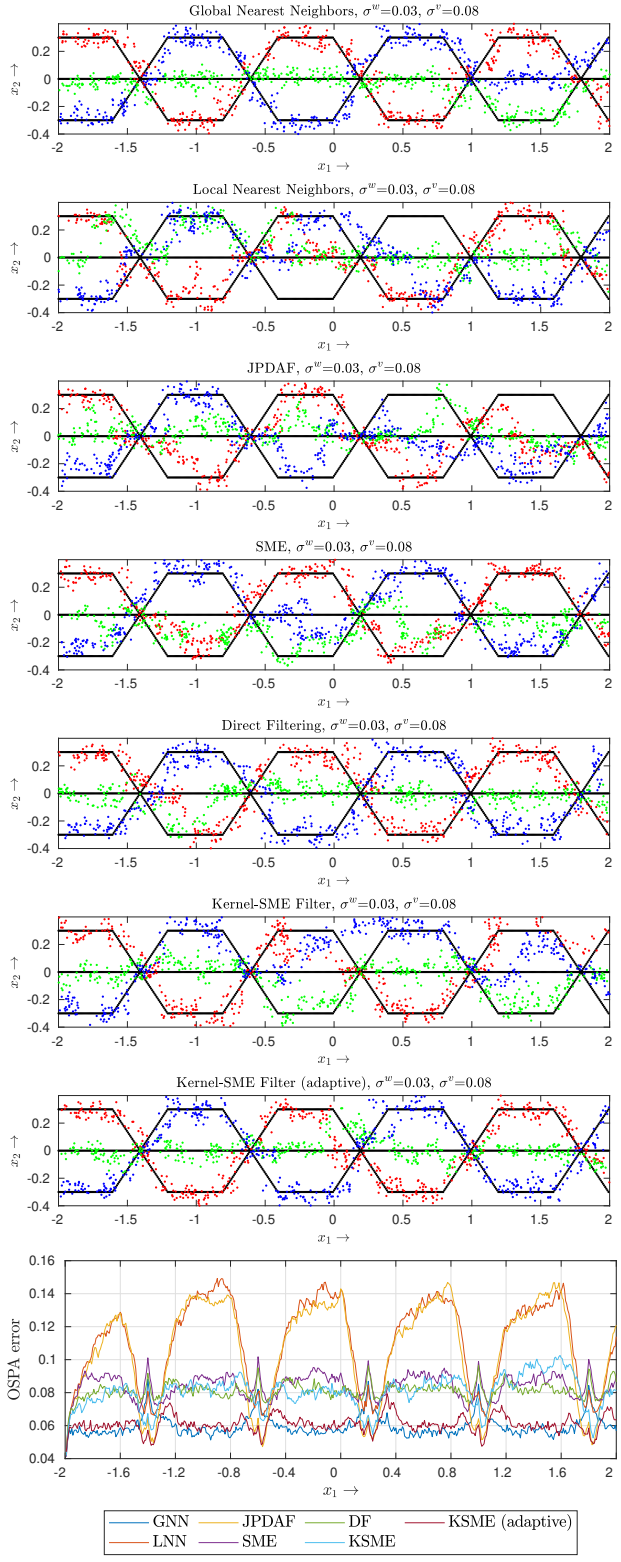
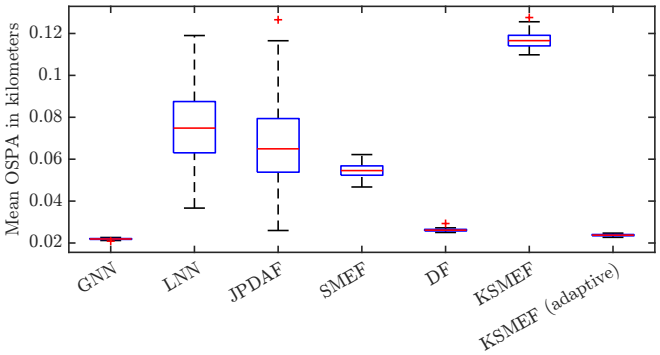
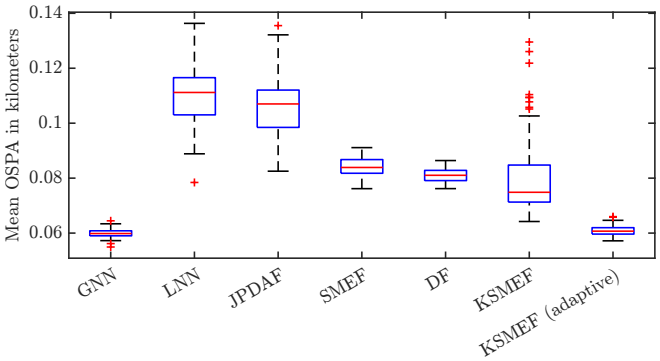


Fig. 2: Considered methods in the high noise scenario of crossing targets with five crossings. The plot at the bottom shows the average OSPA for all locations over all 100 simulation runs along the dimension x_1.



(a) Evaluation results for the low noise scenario.



(b) Evaluation results for the high noise scenario.

Fig. 3: Box plot showing the mean OSPAs (averaged over all time steps) for all runs.

is performed by using the UKF. The augmented directed filtering method based on $P = 37$ deterministic samples is also included. Furthermore, we considered the classical version and our adaptive version of the Kernel-SME filter.

An example of the simulation runs with low measurement noise is shown in Fig. 1 and with high noise in Fig. 2. An evaluation of the algorithms based on 100 simulation runs with the OSPA metric [25] is presented in the box plots in Fig. 3a and Fig. 3b for the two scenarios. While the latter plots allow for a more qualitative evaluation, the former allows for insights concerning the reasons for the quality and the weaknesses the filters showed in this evaluation. In Fig. 3, one can see that the adaptive Kernel-SME filter produces more accurate results than the regular Kernel-SME filter, the direct filtering approach, and the classical SME-based approach. Particularly in Fig. 1, one can see that the JPDAF suffers from its well-known track coalescence problem [26]. Similar effects are observed for the LNN since trackers for different targets start following the same targets and thus use the same measurements, leaving the measurement of one target unconsidered.

Compared with our proposed adaptive Kernel-SME filter, the direct filtering approach provides similar accuracy for the low-noise scenario but fares worse for the high-noise scenario. The only filter that achieves similar (and even slightly superior) accuracy is the GNN. Since

no clutter was considered, the scenario was well-suited to the application of the GNN, and we expect it to perform significantly worse in presence of clutter. Furthermore, with (for similar numbers of targets and measurements) cubic complexity in the number of targets, the GNN results in high run times for high numbers of targets.

V. Conclusion and Outlook

This paper revisited SME-based filter methods for association-free multi-target tracking and provided an extension to this promising paradigm. Due to their low computational complexity, these methods provide a good alternative to the existing association-based algorithms. Our novel adaptive Kernel-SME filter provides a solution for a parameter selection problem of the Kernel-SME filter. The proposed adaptive kernel width takes the uncertainty of the state estimates into account and thus constitutes a sophisticated extension of the Kernel-SME filter. Our extension increases the performance of the original Kernel-SME filter significantly and showed less susceptibility to undesired effects such as track coalescence. Furthermore, the adaptive Kernel-SME filter demonstrates superior performance in comparison with most state-of-the-art filters. One advantage from the practitioner's perspective is that the proposed adaptive kernel width avoids the need for extensive parameter tuning.

Future research will focus on the investigation of the SME-based approaches in scenarios including clutter and missed detection. Moreover, further evaluations in more complex scenarios will be considered. Finally, another extension of the proposed method is to consider anisotropic Gaussian kernels.

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