On Optimal Distributed Kalman Filtering in Non-ideal Situations

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***Abstract*—The distributed processing of measurements and the subsequent data fusion is called Track-to-Track fusion. Although a solution for the Track-to-Track fusion that is equivalent to a central processing scheme has been proposed, this algorithm suffers from strict requirements regarding the local availability of knowledge about utilized models of the remote nodes. By means of simple examples, we investigate the effects of incorrectly assumed models and trace the errors back to a bias, which we derive in closed form. We propose an extension to the exact Track-to-Track fusion algorithm that corrects the bias after arbitrarily many time steps. This new approach yields optimal results when the assumptions about the measurement models are correct and otherwise still provides the exact value for the mean-squared-error matrix. The performance of this algorithm is demonstrated and applications are presented that, e.g., allow the employment of nonlinear filter methods.**

I. Introduction

For the purpose of computing an estimate of an uncertain state, the Bayesian estimation framework provides the means to update estimates with the observed data, to fuse different estimates of the same state, and to account for the temporal evolution of the state by predicting the corresponding estimates. In large-scale networks, it is often desired not to communicate the measurement information of each sensor node to a central station that manages the entire state estimation process. Although multisensor data can be very efficiently processed by means of the information filter [1], which basically is an algebraic reformulation of the Kalman filtering scheme, a frequent and reliable communication is inevitable. Therefore, it is often preferable to compute estimates locally on sensor sites and to interchange them between the nodes in order to fuse them to a global estimate. Forming a global estimate from estimates that only incorporate local sensor data and local prediction steps corresponds to the problem of fusing two tracks, i.e., Track-to-Track fusion (T2TF).

The main difficulty of T2TF manifests itself within the cross-correlations between the local tracks to be fused. If these interdependencies are known, the Bar-Shalom/Campo formulas [2] encompass the best fusion rule. Correlations between locally processed estimates further imply that a local observation not only updates the corresponding local estimate, but also all other estimates that are correlated to the local one. Therefore, establishing an estimation quality that equals a centralized Kalman filter again comes at the cost of full-rate communication. Otherwise, the results of the Bar-Shalom/Campo rule are suboptimal, but in general, still precise.

As sources of correlation, common process noise and common prior information can be identified. The former is due the fact that for a single state, the same system noise is modeled on several sensor nodes, which implies a full correlation between the local noise terms. The latter results from common prior information or double-counting of data, i.e., data that is already included in both local estimates to be fused. Cycles in the network topology can, for example, prevent the common data to be separated from the independent information. In the case of negligible process noise or a full-rate communication, a hierarchical network topology can be employed in order to allow for identifying common information [3], [4] and removing it from the fusion result, e.g., by means of the channel filter [5]. In general, neither the process noise can be ignored nor the sensor sites can communicate at full rate. Instead of striving for an optimal fusion result, suboptimal fusion algorithms, such as Covariance Intersection [6]–[8] or Ellipsoidal Intersection [9], provide conservative estimates that do not underestimate the actual mean-squared-error matrix. These methods yield covariance-consistent, but often less informative fusion results.

An alternative approach is to modify the local estimators in order to keep the tracks decorrelated. For this purpose, the federated Kalman filter [10], [11] alters the system noise in each local prediction step, such that the effect of the common process noise is canceled. If all local estimates can be collected and fused at each time instant, the optimal Kalman filter result can be embodied, otherwise the result is a conservative bound. The federated Kalman filter requires that each node is aware of the total number of nodes participating in the network. Furthermore, cycles must not occur. Koch and Govaers provided an exact distributed Kalman filter algorithm [12]–[14] that employs the same relaxed prediction step as the federated Kalman filter, but also modifies the filtering step by globalizing the likelihood function. After arbitrarily many time steps, the fusion of all local estimates captures exactly the same result as a centralized Kalman filter. This solution to distributed T2TF involves further prerequisites. In order to compute the globalized likelihood, each node must know the sensor models, i.e., measurement mappings and noise terms, of each other node. The difficulty with the globalization becomes apparent when nodes fail or sensor models change, for example, due to linearization.

After elucidating the exact distributed estimation scheme from [12]–[14] in Sec. II, we will analyze the aforementioned issues in Sec. III. Sec. IV provides a calculation of the bias that is introduced to the global estimate when the local models rely on incorrect assumptions. In Sec. V, we will show how to eliminate the bias in order to again compute a consistent fusion result. An evaluation of the proposed concept in Sec. VI and an outlook in Sec. VII conclude this paper.

II. Exact Track-To-Track Fusion algorithm

The exact T2TF algorithm proposed in [12] and [14] adapts both the local prediction and the local filtering step to ensure that the local tracks remain decorrelated. Let

$$\underline{\boldsymbol{x}}^i_{k|k} \sim \mathcal{N}\left(\hat{\underline{x}}^i_{k|k}, \mathbf{C}^{x_i}_{k|k}\right) \tag{1}$$

denote the local estimate of the i-th sensor node with mean vector $\hat{\underline{x}}^i_{k|k}$ and covariance matrix $\mathbf{C}^{x_i}_{k|k}$, where the subscript $k|l$ indicates that observations up to time step l have been incorporated into the state estimate at time step k. If the local estimates (1) are uncorrelated, the global fusion result of N estimates is obtained by the Kalman filter formulas

$$\begin{aligned} \hat{\underline{x}}_{k|k} &= \sum_{i=1}^{N} \mathbf{C}^x_{k|k} \left(\mathbf{C}^{x_i}_{k|k}\right)^{-1} \hat{\underline{x}}^i_{k|k} \text{ and} \\ (\mathbf{C}^x_{k|k})^{-1} &= \sum_{i=1}^{N} \left(\mathbf{C}^{x_i}_{k|k}\right)^{-1} . \end{aligned} \tag{2}$$

In order to initialize the local estimators, the local estimates (1) are globalized by transforming them into $\left(\bar{\underline{x}}^i_{k|k}, \bar{\mathbf{C}}^x_{k|k}\right)$ with

$$\begin{aligned} \bar{\underline{x}}^i_{k|k} =& \bar{\mathbf{C}}^x_{k|k} \left(\mathbf{C}^{x_i}_{k|k}\right)^{-1} \hat{\underline{x}}^i_{k|k} \text{ and} \\ (\bar{\mathbf{C}}^x_{k|k})^{-1} =& \frac{1}{N} \sum_{i=1}^{N} \left(\mathbf{C}^{x_i}_{k|k}\right)^{-1} . \end{aligned} \tag{3}$$

Note that the transformed local estimates share the same globalized covariance matrix $\bar{\mathbf{C}}^x_{k|k}$, they only differ in $\bar{\underline{x}}^i_{k|k}$. Apparently, applying the fusion rule (2) to the globalized estimates (3) again yields the same global fusion result.

The local prediction steps are carried out by means of a relaxed system model, i.e., the parameters of $\left(\bar{\underline{x}}^i_{k+1|k}, \bar{\mathbf{C}}^x_{k+1|k}\right)$ are obtained by

$$\begin{aligned} \bar{\underline{x}}^i_{k+1|k} =& \mathbf{A}_k \bar{\underline{x}}^i_{k|k} \text{ and} \\ \bar{\mathbf{C}}^x_{k+1|k} =& \mathbf{A}_k \bar{\mathbf{C}}^x_{k|k} \mathbf{A}_k^{\mathrm{T}} + N \mathbf{C}^w_k , \end{aligned} \tag{4}$$

where $\mathbf{C}^w_k$ is the covariance matrix of a white Gaussian system noise. The local predictors employ scaled versions of $\mathbf{C}^w_k$, which complies with the local prediction models of the federated Kalman filter [10]. The relaxed prediction step obviously preserves the equality of the local covariance matrices.

Fusing estimates locally with observations will result in different covariance matrices and therefore, the tracks will not remain decorrelated. Koch presents two solutions to this issue [12], [14]: Either the filter step is performed locally according to the Kalman filter principle and the estimates are globalized afterwards by means of (3) or a globalized likelihood function is applied. In the latter case, the filter step is given by

$$\begin{aligned} \bar{\underline{x}}^i_{k|k} =& \bar{\mathbf{C}}^x_{k|k} \left(\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1} \bar{\underline{x}}^i_{k|k-1} + \left(\mathbf{H}^i_k\right)^{\mathrm{T}} \left(\mathbf{C}^{v_i}_k\right)^{-1} \underline{z}^i_k \right) , \\ \bar{\mathbf{C}}^x_{k|k} =& \left(\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1} + \left(\bar{\mathbf{C}}^v_k\right)^{-1} \right)^{-1} \text{ with} \\ \left(\bar{\mathbf{C}}^v_k\right)^{-1} =& \frac{1}{N} \sum_{j \in \bar{\mathcal{M}}_k} \left(\mathbf{H}^j_k\right)^{\mathrm{T}} \left(\mathbf{C}^{v_j}_k\right)^{-1} \mathbf{H}^j_k , \end{aligned} \tag{5}$$

where $\bar{\mathcal{M}}_k$ contains the indices of those sensor nodes that obtain measurements $\underline{z}^i_k$ at time step k.

It is straightforward to show that the convex combination, i.e., fusion (2), of the globalized tracks yields the central fusion result after all globalized prediction and filter steps. The following example will explicate the high attainable accuracy of this approach.

Example II.1 The idea of this example is to reconstruct the route of an object in 2D space with multiple randomly placed sensors.

We assume the sensors to have a small internal storage, limited computational power, and no communication capabilities. Many of these sensors are randomly placed in a 2D area and locally estimate the state of an object. After a certain time, the sensor data is collected and the result is evaluated at a powerful workstation.

As the internal storages of the sensors are limited, the measurements, estimates etc. cannot be saved separately, but must be stored recursively or in compressed form. To do so, the sensors recursively calculate local estimates by means of the exact T2TF and store them every 10th time step in order to allow a reconstruction of the object track later on.

Let the number of sensors be $N = 100$ and the number of time steps also be 100. The system model is given as a disturbed rotational motion model

$$\underline{\boldsymbol{x}}_{k+1} = \begin{pmatrix} 1 & \frac{\sin(\omega)}{\omega} & 0 & -\frac{1-\cos(\omega)}{\omega} \\ 0 & \cos(\omega) & 0 & -\sin(\omega) \\ 0 & \frac{1-\cos(\omega)}{\omega} & 1 & \frac{\sin(\omega)}{\omega} \\ 0 & \sin(\omega) & 0 & \cos(\omega) \end{pmatrix} \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{w}}_k \tag{6}$$

with $\omega = 0.13$ and

$$\underline{\boldsymbol{w}}_k \sim \mathcal{N}\left(\underline{0}, \begin{pmatrix} 0.8 & 0 & 0.1 & 0.2 \\ 0 & 0.3 & 0 & 0 \\ 0.1 & 0 & 0.7 & 0 \\ 0.2 & 0 & 0 & 0.4 \end{pmatrix}\right) . \tag{7}$$

The measurement model is

$$\underline{\boldsymbol{z}}^i_k = \mathbf{H}^i_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}^i_k , \tag{8}$$

with $\mathbf{H}^i_k = \mathbf{I}$ and

$$\underline{\boldsymbol{v}}^i_k \sim \mathcal{N}\left(\underline{0}, \begin{pmatrix} 1 & 0.4 & 0 & 0 \\ 0.4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0.25 \\ 0 & 0 & 0.25 & 2 \end{pmatrix}\right) . \tag{9}$$

The tracking results of one run of this scenario are given in Figure 1. As it can be seen, the assumed route matches the real route almost perfectly, which is straightforward as

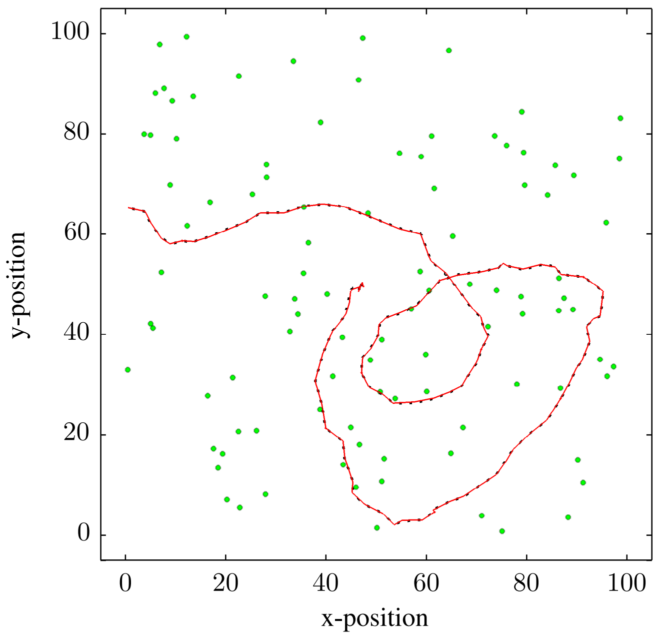

Figure 1. The tracking results of Example II.1. The green points mark the sensor positions, the black dotted path is the real route, and the red path indicates the position estimated by the exact T2TF algorithm at different time steps. Actually, the position estimation is only available each 10th time step. For an easier evaluation, however, we also added the fused results in between these time steps.

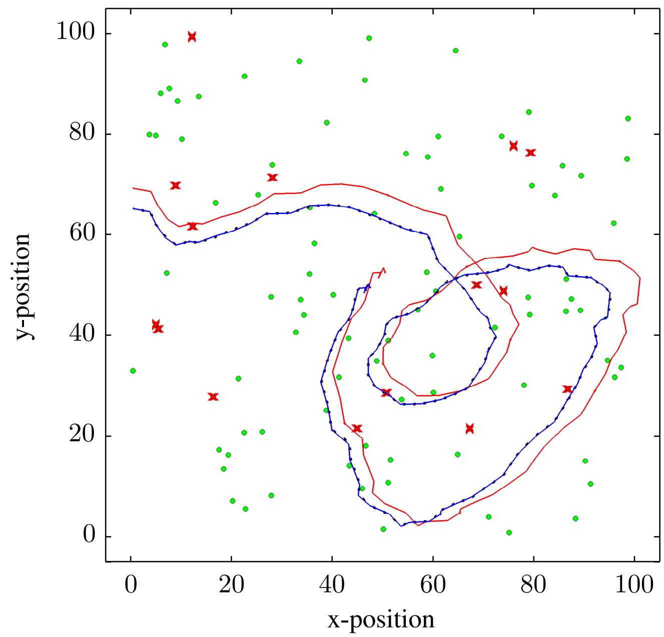

Figure 2. The tracking results of Example III.1. In addition to Figure 1, 15 defect sensors are crossed out and the BLUE solution, which equals the centralized Kalman Filter result, is given as a blue path.

the exact T2TF algorithm equals the best linear unbiased estimator (BLUE) and all sensors obtain good-quality, distance-independent measurements at each time step.

The average error over the available estimates is given by

	BLUE	Exact T2TF
x	0.104	0.104
y	0.07	0.07

.

Applying the exact T2TF formulas not only requires knowledge about the total number of participating nodes, but also exact knowledge about the sensor models and measurement frequencies, as can be seen in (5). The remainder of this paper discusses the situation when reality does not meet the assumptions, i.e., when nodes are not aware of the other nodes' behavior and cannot compute the exact globalized likelihood.

III. Motivation and Problem Statement

Finding the optimal sensor network fusion algorithm remains an open topic in fusion theory. Different approaches such as the Bar-Shalom/Campo formulas or the Channel Filter provide reliable estimations when the common prior information, respectively the process noise, is negligible or a full rate communication is employed. However, they fail to provide consistent[1] estimates when the assumptions are not met. This leads to global estimates whose precision and quality is hard to determine and in general is worse than the central estimate.

Although the exact T2TF algorithm provides the possibility of exact distributed fusion, it can rarely be applied since the assumptions on the global knowledge are restrictive. In particular, in large sensor networks it is hard to guarantee a fixed number of working, respectively defective, sensor nodes. A naïve approach is to give a lower bound for the number of functioning sensors or estimate this number in order to nearly obtain the exact estimate. Both ideas hardly work with the exact T2TF algorithm as it is shown in the next example.

Example III.1 We assume the models to be the same as in Example II.1. In order to make the scenario more realistic, we let the probability of sensor failure be 0.2. This quota is assumed to be known so that the globalized covariance matrix (5) is determined corresponding to the failure rate. Nevertheless, as the exact number of sensor failures is not known before the scenario starts, it is possible that the number of functioning sensors is under- or overestimated.

As it can be seen in Figure 2, the estimate of the exact T2TF algorithm is biased when this happens. This is caused by the difference between the assumed number of defect sensors $100 * 0.2 = 20$ and the real number of defect sensors that is only 15 in this run. The average of the rooted error of each 10th estimate is obtained by

	BLUE	Exact T2TF (biased)
x	0.123	3.833
y	0.138	2.687

.

As it becomes obvious by comparing the results of this example with those of Example II.1, the average error of the centralized BLUE filter is higher when the sensors are defect. Even more remarkable is the increase in the average error of the exact T2TF algorithm. Although less sensors than assumed are defect and therefore, it is reasonable to expect the system to perform well, the estimate is biased.

Besides the rate of failure, sensor networks that consist of many nodes are often established as the estimation quality of many distributed, low-quality sensors is assumed to be better than one high-quality sensor. This, however, usually implies that the local measurement quality depends on the distance

[1] The difference between the real mean-squared-error matrix and the calculated covariance matrix is not positive-semidefinite, which means that the error is underestimated.

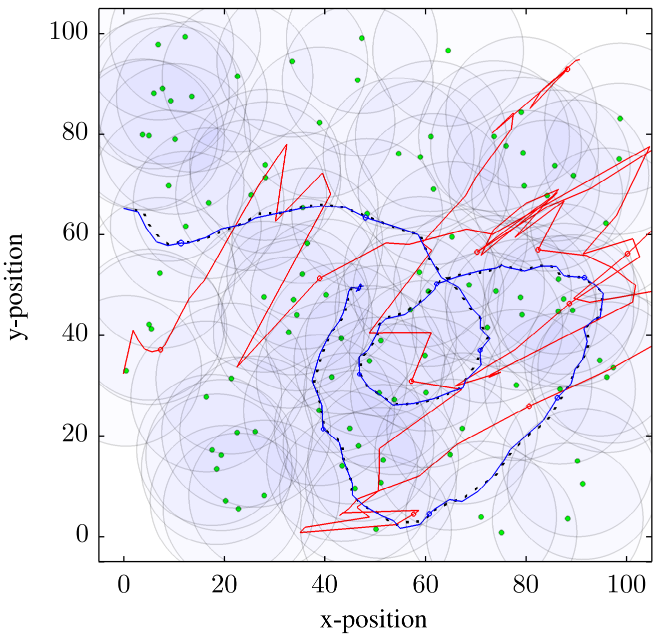

Figure 3. The tracking results of Example III.2. In addition to Figure 1, the sensors obtain measurement only in a limited area that is indicated by the transparent blue circles.

between sensor and target or at least means that not all sensors detect the target at once with equal measurement uncertainties.

Example III.2 Again, we suppose the configuration to be the same as in Example II.1. Instead of assuming a probability of failure, the sensor range is limited by 15 in this example. The most meaningful assumptions seems to be to calculate the globalized likelihood by help of the expected number of sensors that are simultaneously detecting the object. The derivation of this value is out of scope of this paper, but is given in our scenario by approximately 6.3. Therefore, the time-independent, globalized likelihood is obtained by

$$\left(\bar{\mathbf{C}}_k^v\right)^{-1} = 6.3 \cdot (\mathbf{H}_k)^T \left(\mathbf{C}_k^v\right)^{-1} \mathbf{H}_k \;\forall k \in \{1, \ldots, 100\} \,. \tag{10}$$

As expected, it becomes clear from Figure 3 that the exact T2TF algorithm is not working in this scenario. Due to the time-dependent true globalized likelihood that differs considerably from the assumed globalized likelihood, the results are biased. Compared to Example III.1, this bias is significantly higher as the globalized likelihood can be massively under- or overestimated.

By taking the examples and the flaws of the existing algorithms into account when the restrictive assumptions are not met, it is hard to establish a distributed sensor network fusion strategy that gives guarantees about the estimation uncertainty. In the following, the cause for the inconsistency, respectively the bias, of the exact T2TF algorithm is derived and an extension is proposed that guarantees consistency when the models cannot be predetermined.

IV. Bias Calculation

In this section, we investigate the global estimate that is obtained by fusing all local estimates. In particular, we determine the bias of this estimate when the assumptions about the local measurement uncertainties are not met.

In addition to the variables (3) from Sec. II, we consider the fused (global) estimate that is obtained by

$$\begin{aligned}\tilde{\underline{x}}_k &= \tilde{\mathbf{C}}_k^x \left(\sum_{i=1}^{N} \left(\bar{\mathbf{C}}_k^x\right)^{-1} \bar{\underline{x}}_k^i\right) = \frac{1}{N}\sum_{i=1}^{N} \bar{\underline{x}}_k^i \text{ and} \\ \tilde{\mathbf{C}}_k^x &= \left(N\left(\bar{\mathbf{C}}_k^x\right)^{-1}\right)^{-1} = \frac{1}{N}\bar{\mathbf{C}}_k^x\end{aligned} \tag{11}$$

and the true globalized likelihood

$$\left(\tilde{\mathbf{C}}_k^v\right)^{-1} = \frac{1}{N}\sum_{i\in\tilde{\mathcal{M}}_k} \left(\mathbf{H}_k^i\right)^{\mathrm{T}} \left(\mathbf{C}_k^{v_i}\right)^{-1} \mathbf{H}_k^i \,, \tag{12}$$

where $\tilde{\mathcal{M}}_k$ contains the indices of all sensor nodes that have processed a measurement at time step k. The true globalized likelihood is the sum of actually used measurement models, where the assumed globalized likelihood $(\bar{\mathbf{C}}_k^v)^{-1}$ from equation (5) is the likelihood that is utilized in the local sensors to calculate the estimates.

By taking (11) into account, the fused mean is obtained by

$$\begin{aligned}\tilde{\underline{x}}_{k|k} &= \frac{1}{N}\bar{\mathbf{C}}_{k|k}^x\left(\sum_{i=1}^{N}\left(\bar{\mathbf{C}}_{k|k-1}^x\right)^{-1}\bar{\underline{x}}_{k|k-1}^i + \sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{H}_k^j\right)^{\mathrm{T}}\left(\mathbf{C}_k^{v_j}\right)^{-1}\underline{z}_k^j\right) \\ &= \frac{1}{N}\bar{\mathbf{C}}_{k|k}^x\left(N\left(\bar{\mathbf{C}}_{k|k-1}^x\right)^{-1}\tilde{\underline{x}}_{k|k-1} + \sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{H}_k^j\right)^{\mathrm{T}}\left(\mathbf{C}_k^{v_j}\right)^{-1}\underline{z}_k^j\right).\end{aligned} \tag{13}$$

By help of (5), the true state is given as

$$\begin{aligned}\underline{\boldsymbol{x}}_k &= \frac{N}{N}\bar{\mathbf{C}}_{k|k}^x\left(\bar{\mathbf{C}}_{k|k}^x\right)^{-1}\underline{\boldsymbol{x}}_k \\ &= \frac{1}{N}\bar{\mathbf{C}}_{k|k}^x\left(N\left(\bar{\mathbf{C}}_{k|k-1}^x\right)^{-1}\underline{\boldsymbol{x}}_k + N\left(\bar{\mathbf{C}}_k^v\right)^{-1}\underline{\boldsymbol{x}}_k\right) \,.\end{aligned} \tag{14}$$

Therefore, we obtain for the difference between the estimated and the true state

$$\underline{\boldsymbol{x}}_k^d = \tilde{\underline{x}}_{k|k} - \underline{\boldsymbol{x}}_k = \frac{1}{N}\bar{\mathbf{C}}_{k|k}^x\left(\mathbf{R}_k + \sum_{j\in\tilde{\mathcal{M}}_k}\mathbf{S}_{k|k-1}^j\right) \,, \tag{15}$$

with

$$\begin{aligned}\mathbf{R}_k &= N\left(\bar{\mathbf{C}}_{k|k-1}^x\right)^{-1}\left(\tilde{\underline{x}}_{k|k-1} - \underline{\boldsymbol{x}}_k\right) \text{ and} \\ \mathbf{S}_k^j &= \left(\left(\mathbf{H}_k^j\right)^{\mathrm{T}}\left(\mathbf{C}_k^{v_j}\right)^{-1}\underline{z}_k^j - \frac{N}{|\tilde{\mathcal{M}}_k|}\left(\bar{\mathbf{C}}_k^v\right)^{-1}\underline{\boldsymbol{x}}_k\right) \,.\end{aligned} \tag{16}$$

In order to obtain the true covariance of $\tilde{\underline{x}}_{k|k}$ based on the predicted estimates and measurements, we evaluate $\mathrm{E}\left\{\underline{\boldsymbol{x}}_k^d\left(\underline{\boldsymbol{x}}_k^d\right)^{\mathrm{T}}\right\}$, which is given by

$$\begin{aligned}&\frac{1}{N^2}\bar{\mathbf{C}}_{k|k}^x\mathrm{E}\Bigg\{\mathbf{R}_k\left(\mathbf{R}_k\right)^{\mathrm{T}} + \sum_{i\in M_k}\sum_{j\in M_k}\mathbf{S}_k^i\left(\mathbf{S}_k^j\right)^{\mathrm{T}} + \\ &\mathbf{R}_k\sum_{i\in M_k}\left(\mathbf{S}_k^i\right)^{\mathrm{T}} + \sum_{i\in M_k}\mathbf{S}_k^i\left(\mathbf{R}_k\right)^{\mathrm{T}}\Bigg\}\left(\bar{\mathbf{C}}_{k|k}^x\right)^{\mathrm{T}} \,.\end{aligned} \tag{17}$$

Let $(\cdot)^2$ be a short version of the matrix $(\cdot)(\cdot)^T$, then the product $\mathbf{R}_k\left(\mathbf{R}_k\right)^{\mathrm{T}}$ equals

$$N^2\left(\bar{\mathbf{C}}_{k|k-1}^x\right)^{-1}\mathrm{E}\left\{\left(\tilde{\underline{x}}_{k|k-1} - \underline{\boldsymbol{x}}_k\right)^2\right\}\left(\bar{\mathbf{C}}_{k|k-1}^x\right)^{-\mathrm{T}} \,, \tag{18}$$

which is simplified to

$$\mathbf{R}_k(\mathbf{R}_k)^{\mathrm{T}}=N^2\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1}\tilde{\mathbf{C}}^x_{k|k-1}\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-\mathrm{T}}=N\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1} \tag{19}$$

when the covariance matrix estimation $\tilde{\mathbf{C}}^x_{k|k-1}$ is correct.

With $\underline{z}^j_k = \mathbf{H}^j_k \underline{\boldsymbol{x}}_k + \underline{\boldsymbol{v}}^j_k$, the term $\mathbf{S}^j_k$ is transformed to

$$\left(\mathbf{H}^j_k\right)^{\mathrm{T}}\left(\mathbf{C}^{v_j}_k\right)^{-1}\underline{\boldsymbol{v}}^j_k + \left(\left(\mathbf{H}^j_k\right)^{\mathrm{T}}\left(\mathbf{C}^{v_j}_k\right)^{-1}\mathbf{H}^j_k - \frac{N}{|\tilde{\mathcal{M}}_k|}\left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)\underline{\boldsymbol{x}}_k \ . \tag{20}$$

Thus, the sum $\sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{S}^j_k\right)^{\mathrm{T}}$ equals

$$\sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{H}^j_k\right)^{\mathrm{T}}\left(\mathbf{C}^{v_j}_k\right)^{-1}\underline{\boldsymbol{v}}^j_k + \left(N\left(\tilde{\mathbf{C}}^v_k\right)^{-1} - N\left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)\underline{\boldsymbol{x}}_k \ . \tag{21}$$

The measurement noise $\underline{\boldsymbol{v}}^j_k$ is independent of $\underline{\boldsymbol{x}}_k$ and of $\underline{\boldsymbol{v}}^i_k$ for $i \neq j$. Therefore, the expected value of the product $\mathrm{E}\left\{\sum_{i,j\in\tilde{\mathcal{M}}_k}\mathbf{S}^i_k\left(\mathbf{S}^j_k\right)^{\mathrm{T}}\right\}$ is obtained by

$$\sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{H}^j_k\right)^{\mathrm{T}}\left(\mathbf{C}^{v_j}_k\right)^{-1}\mathbf{H}^j_k + \left(N\left(\tilde{\mathbf{C}}^v_k\right)^{-1} - N\left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)\cdot \mathrm{E}\left\{\underline{\boldsymbol{x}}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\right\}\left(N\left(\tilde{\mathbf{C}}^v_k\right)^{-1} - N\left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)^{\mathrm{T}} . \tag{22}$$

For $(\bar{\mathbf{C}}^v_k)^{-1} = (\tilde{\mathbf{C}}^v_k)^{-1}$, this is simplified according to (12) to $N(\tilde{\mathbf{C}}^v_k)^{-1}$.

By help of (21), we calculate the cross-correlation $\mathrm{E}\left\{\mathbf{R}_k\sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{S}^j_k\right)^{\mathrm{T}}\right\}$ up to

$$\mathrm{E}\left\{\mathbf{R}_k\left(\sum_{j\in\tilde{\mathcal{M}}_k}\left(\mathbf{H}^j_k\right)^{\mathrm{T}}\left(\mathbf{C}^{v_j}_k\right)^{-1}\underline{\boldsymbol{v}}^j_k\right)^T + \mathbf{R}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\left(N\left(\tilde{\mathbf{C}}^v_k\right)^{-1} - N\left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)^{\mathrm{T}}\right\} . \tag{23}$$

By taking the definition of $\mathbf{R}_k$ from equation (16) and the independence of $\underline{\boldsymbol{x}}_k$ and $\underline{\boldsymbol{v}}^j_k$ into account, this is simplified to

$$N^2\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1}\mathrm{E}\left\{\underline{\tilde{x}}_{k|k-1}\underline{\boldsymbol{x}}_k^{\mathrm{T}} - \underline{\boldsymbol{x}}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\right\}\left(\left(\tilde{\mathbf{C}}^v_k\right)^{-1} - \left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)^{\mathrm{T}} . \tag{24}$$

However, for $(\bar{\mathbf{C}}^v_k)^{-1} = (\tilde{\mathbf{C}}^v_k)^{-1}$, the cross-correlation is zero.

Thus, when the assumed globalized likelihood covariance matrix $(\bar{\mathbf{C}}^v_k)^{-1}$ equals the actual globalized likelihood covariance matrix and $\tilde{\mathbf{C}}^x_{k|k-1}$ is a correct estimation of the predicted error covariance matrix, the error covariance matrix $\mathrm{E}\left\{\underline{\boldsymbol{x}}^d_k\left(\underline{\boldsymbol{x}}^d_k\right)^{\mathrm{T}}\right\}$ of the filtered global estimation is

$$\frac{1}{N^2}\bar{\mathbf{C}}^x_{k|k}\left(N\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1} + N\left(\bar{\mathbf{C}}^v_k\right)^{-1}\right)\left(\bar{\mathbf{C}}^x_{k|k}\right)^{\mathrm{T}} = \frac{1}{N}\bar{\mathbf{C}}^x_{k|k} \ , \tag{25}$$

which equals the error estimation $\tilde{\mathbf{C}}^x_{k|k}$. This is a straightforward result since the presented procedure is optimal in the Kalman filter sense. However, the derivation of the error covariance matrix becomes interesting when the globally assumed measurement covariance matrix does not equal the actual one, i.e., $(\bar{\mathbf{C}}^v_k)^{-1} \neq (\tilde{\mathbf{C}}^v_k)^{-1}$. When $\tilde{\mathbf{C}}_{k|k-1}$ is the correct error covariance matrix, (24) is given as

$$\mathrm{E}\left\{\underline{\tilde{x}}_{k|k-1}\underline{\boldsymbol{x}}_k^{\mathrm{T}} - \underline{\boldsymbol{x}}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\right\} = -\tilde{\mathbf{C}}_{k|k-1} + \mathrm{E}\left\{\underline{\tilde{x}}_{k|k-1}\underline{\tilde{x}}_{k|k-1}^{\mathrm{T}} - \underline{\boldsymbol{x}}_k\cdot\underline{\tilde{x}}_{k|k-1}^{\mathrm{T}}\right\} = -\tilde{\mathbf{C}}_{k|k-1} + \mathrm{E}\left\{\underline{\tilde{x}}_{k|k-1} - \underline{\boldsymbol{x}}_k\right\}\underline{\tilde{x}}_{k|k-1}^{\mathrm{T}} = -\tilde{\mathbf{C}}_{k|k-1} \ . \tag{26}$$

Then, the error covariance matrix of the globally fused result $\mathrm{E}\left\{\underline{\boldsymbol{x}}^d_k\left(\underline{\boldsymbol{x}}^d_k\right)^{\mathrm{T}}\right\}$ is obtained with (19), (22) and (24) as

$$\bar{\mathbf{C}}^x_{k|k}\left(\frac{1}{N}\left(\bar{\mathbf{C}}^x_{k|k-1}\right)^{-1} + \frac{1}{N}\left(\bar{\mathbf{C}}^v_k\right)^{-1} + \frac{1}{N}\Delta\mathbf{C}^v_k + \Delta\mathbf{C}^v_k\cdot \mathrm{E}\left\{\underline{\boldsymbol{x}}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\right\}\left(\Delta\mathbf{C}^v_k\right)^{\mathrm{T}} - \frac{1}{N}\Delta\mathbf{C}^v_k - \frac{1}{N}\left(\Delta\mathbf{C}^v_k\right)^{\mathrm{T}}\right)\left(\bar{\mathbf{C}}^x_{k|k}\right)^{\mathrm{T}} \tag{27}$$

with

$$\Delta\mathbf{C}^v_k = \left(\tilde{\mathbf{C}}^v_k\right)^{-1} - \left(\bar{\mathbf{C}}^v_k\right)^{-1} \ . \tag{28}$$

This equation is simplified to

$$\frac{1}{N}\bar{\mathbf{C}}^x_{k|k} + \frac{1}{N}\bar{\mathbf{C}}^x_{k|k}\left(\Delta\mathbf{C}^v_k\mathrm{E}\left\{\underline{\boldsymbol{x}}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\right\} - \mathbf{I}\right)\left(\Delta\mathbf{C}^v_k\right)^{\mathrm{T}}\left(\bar{\mathbf{C}}^x_{k|k}\right)^{\mathrm{T}} . \tag{29}$$

It is obvious from (29) that the positive-definite term $\mathrm{E}\left\{\underline{\boldsymbol{x}}_k\underline{\boldsymbol{x}}_k^{\mathrm{T}}\right\}$ denotes the bias and takes values of arbitrary size and thus, cannot be bounded by a state-independent covariance matrix.

Therefore, when at least one sensor in the network stops working, but also when an additional sensor is added to the sensor network or the assumed globalized likelihood does not exactly match the real one, then the true error covariance matrix can be larger than the estimated one and thus, the estimation is inconsistent.

V. Self-adapting exact T2TF

The inconsistency of the presented exact T2TF algorithm, when the globally assumed parameters are not exactly met, arises mainly from the derived bias (27). In the following, we propose a procedure to eliminate this bias and to guarantee consistent results even when the globally assumed parameters are not exactly met.

As the presented T2TF algorithm allows an independent calculation of the local estimates, it is not surprising that the bias is caused by an incorrect combination of the globalized local estimates to the global estimate. Based on the actually utilized measurement models, a matrix to correct the bias that is caused by the incorrectly assumed measurement models is calculated recursively. For that purpose, we distinguish between initialization, prediction, and filtering.

A. Initialization

We assume that the nodes with indices in $\tilde{\mathcal{M}}_1$ have performed consistent measurements[2] and have fused them

[2]The local measurement uncertainties $\mathbf{C}^{v_i}_1$ have not been underestimated.

according to equation (5) to there own estimates. When $\tilde{\mathcal{M}}_1 \neq \bar{\mathcal{M}}_1$, the assumed globalized measurement covariance matrix $(\bar{\mathbf{C}}_1^v)^{-1}$ does not match the real one and thus, by applying the standard exact T2TF algorithm, the global estimate is biased and inconsistent.

Therefore, we define

$$\boldsymbol{\Delta}_{1|1}^x = \hat{\mathbf{C}}_{1|1}^x \left(\bar{\mathbf{C}}_{1|1}^x\right)^{-1} \text{with } \hat{\mathbf{C}}_{1|1}^x = \left(\left(\bar{\mathbf{C}}_{1|0}^x\right)^{-1} + \left(\tilde{\mathbf{C}}_1^v\right)^{-1}\right)^{-1} . \tag{30}$$

The bias is eliminated by scaling the global estimate $\tilde{\underline{x}}_{1|1}$ by $\boldsymbol{\Delta}_{1|1}^x$. The real error matrix is obtained by simple algebraic operations based on the difference between the scaled estimate and the true random variable

$$\underline{\boldsymbol{x}}_{1|1}^{d'} = \boldsymbol{\Delta}_{1|1}^x \tilde{\underline{x}}_{1|1} - \underline{\boldsymbol{x}}_1 = \frac{1}{N} \boldsymbol{\Delta}_{1|1}^x \sum_{i=1}^{N} \bar{\underline{x}}_{1|1}^i - \underline{\boldsymbol{x}}_1 \tag{31}$$

as

$$\tilde{\mathbf{P}}_{1|1}^x = \mathrm{E}\left\{ \underline{\boldsymbol{x}}_{1|1}^{d'} \left(\underline{\boldsymbol{x}}_{1|1}^{d'}\right)^T \right\} = \frac{1}{N} \hat{\mathbf{C}}_{1|1}^x \ . \tag{32}$$

B. Prediction

The bias elimination of the predicted estimate is reduced to the bias elimination of the initialization/filtering step by defining $\boldsymbol{\Delta}_{k+1|k}^x = \mathbf{A} \boldsymbol{\Delta}_{k|k}^x (\mathbf{A})^{-1}$. We obtain for the difference term

$$\begin{aligned} \underline{\boldsymbol{x}}_{k+1|k}^{d'} &= \boldsymbol{\Delta}_{k+1|k}^x \tilde{\underline{x}}_{k+1|k} - \underline{\boldsymbol{x}}_{k+1} \\ &= \boldsymbol{\Delta}_{k+1|k}^x \mathbf{A} \tilde{\underline{x}}_{k|k} - \mathbf{A} \underline{\boldsymbol{x}}_k - \underline{\boldsymbol{w}}_k \\ &= \mathbf{A} \left(\boldsymbol{\Delta}_{k|k}^x \tilde{\underline{x}}_{k|k} - \underline{\boldsymbol{x}}_k \right) - \underline{\boldsymbol{w}}_k \end{aligned} \tag{33}$$

and so, the error covariance matrix, considering the independence between the noise term $\underline{\boldsymbol{w}}_k$ and the state $\underline{\boldsymbol{x}}_k$, is given by

$$\begin{aligned} \tilde{\mathbf{P}}_{k+1|k}^x &= \mathrm{E}\left\{ \underline{\boldsymbol{x}}_{k+1|k}^{d'} \left(\underline{\boldsymbol{x}}_{k+1|k}^{d'}\right)^{\mathrm{T}} \right\} \\ &= \mathbf{A} \mathrm{E}\left\{ \left(\boldsymbol{\Delta}_{k|k}^x \tilde{\underline{x}}_{k|k} - \underline{\boldsymbol{x}}_k \right)^2 \right\} (\mathbf{A})^{\mathrm{T}} + \mathbf{C}_k^w \\ &= \mathbf{A} \tilde{\mathbf{P}}_{k|k}^x (\mathbf{A})^{\mathrm{T}} + \mathbf{C}_k^w \ . \end{aligned} \tag{34}$$

C. Filtering

The most of what has to be taken care of to eliminate the bias in the filter step has been considered in the initialization paragraph V-A. In a more general form, we define the matrix

$$\boldsymbol{\Delta}_{k|k}^x = \hat{\mathbf{C}}_{k|k}^x \left(\bar{\mathbf{C}}_{k|k}^x \right)^{-1} \tag{35}$$

with

$$\hat{\mathbf{C}}_{k|k}^x = \left(\left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-1} \left(\boldsymbol{\Delta}_{k|k-1}^x \right)^{-1} + \left(\tilde{\mathbf{C}}_k^v \right)^{-1} \right)^{-1} \ . \tag{36}$$

It remains to derive the error covariance matrix for $\boldsymbol{\Delta}_{k|k-1}^x \neq \mathbf{I}$ and $\tilde{\mathbf{P}}_{k|k-1}^x = \mathrm{E}\left\{ \underline{\boldsymbol{x}}_{k|k-1}^{d'} \left(\underline{\boldsymbol{x}}_{k|k-1}^{d'} \right)^{\mathrm{T}} \right\} \neq \tilde{\mathbf{C}}_{k|k-1}^x$. We define $\underline{\boldsymbol{x}}_{k|k}^{d'}$ analogously to (33) and split it up into

$$\frac{1}{N} \hat{\mathbf{C}}_{k|k}^x \left(\sum_{i=1}^{N} \left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-1} \bar{\underline{x}}_{k|k-1}^i + \sum_{j \in \tilde{\mathcal{M}}_k} \left(\mathbf{H}_k^j \right)^{\mathrm{T}} \left(\mathbf{C}_k^{v_j} \right)^{-1} \underline{z}_k^j \right) - \frac{1}{N} \hat{\mathbf{C}}_{k|k}^x \left(N \left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-1} \left(\boldsymbol{\Delta}_{k|k-1}^x \right)^{-1} \underline{\boldsymbol{x}}_k + N \left(\tilde{\mathbf{C}}_k^v \right)^{-1} \underline{\boldsymbol{x}}_k \right) . \tag{37}$$

Equation (37), however, equals

$$\frac{1}{N} \hat{\mathbf{C}}_{k|k}^x \left(\left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-1} \left(\sum_{i=1}^{N} \bar{\underline{x}}_{k|k-1}^i - N \left(\boldsymbol{\Delta}_{k|k-1}^x \right)^{-1} \underline{\boldsymbol{x}}_k \right) + \sum_{j \in \tilde{\mathcal{M}}_k} \left(\mathbf{H}_k^j \right)^{\mathrm{T}} \left(\mathbf{C}_k^{v_j} \right)^{-1} \underline{z}_k^j - N \left(\tilde{\mathbf{C}}_k^v \right)^{-1} \underline{\boldsymbol{x}}_k \right) \tag{38}$$

and can be further simplified to

$$\frac{1}{N} \hat{\mathbf{C}}_{k|k}^x \left(N \left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-1} \left(\boldsymbol{\Delta}_{k|k-1}^x \right)^{-1} \left(\boldsymbol{\Delta}_{k|k-1}^x \tilde{\underline{x}}_{k|k-1} - \underline{\boldsymbol{x}}_k \right) + \sum_{j \in \tilde{\mathcal{M}}_k} \left(\left(\mathbf{H}_k^j \right)^{\mathrm{T}} \left(\mathbf{C}_k^{v_j} \right)^{-1} \underline{z}_k^j - \frac{N}{|\tilde{\mathcal{M}}_k|} \left(\tilde{\mathbf{C}}_k^v \right)^{-1} \underline{\boldsymbol{x}}_k \right) \right) . \tag{39}$$

Finally, the true error covariance matrix is given as

$$\tilde{\mathbf{P}}_{k|k}^x = \frac{1}{N^2} \hat{\mathbf{C}}_{k|k}^x \left(N^2 \left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-1} \left(\boldsymbol{\Delta}_{k|k-1}^x \right)^{-1} \tilde{\mathbf{P}}_{k|k-1}^x \cdot \left(\boldsymbol{\Delta}_{k|k-1}^x \right)^{-\mathrm{T}} \left(\bar{\mathbf{C}}_{k|k-1}^x \right)^{-\mathrm{T}} + N \left(\tilde{\mathbf{C}}_k^v \right)^{-1} \right) \left(\hat{\mathbf{C}}_{k|k}^x \right)^{\mathrm{T}} . \tag{40}$$

It is worth mentioning that in general $\boldsymbol{\Delta}_{k|k-1}^x \bar{\mathbf{C}}_{k|k-1}^x \neq \tilde{\mathbf{P}}_{k|k-1}^x$ holds and the error covariance bound $\tilde{\mathbf{P}}_{k|k}^x$ is no longer globally optimal as some information has been lost due to the processing with the incorrectly assumed measurement models. Nevertheless, the proposed extension to the exact T2TF offers system designers a flexible and efficient method for the sensor network fusion, when the local estimates are collected in a sink and the locally available estimates are of marginal interest.

VI. Evaluation

The main challenge in applying the proposed algorithm is the transmission of the actually utilized measurement models and the computational effort in order to obtain the bias correction matrix. As the focus of this algorithm is not laid on obtaining high quality estimates in the local nodes, the application is especially meaningful in sensor networks with a distinct fusion center and therefore, the computational burden should be usually negligible. Nevertheless, the demand for communication is often a limiting factor in sensor networks and should be minimized. Thus, we discuss the application of the extended exact T2TF algorithm in different scenarios and propose ideas to reduce the necessary communication effort that is necessary to receive the actually utilized measurement models.

In simple scenarios, where the utilized measurement models are communicated (time delayed), the application of the proposed extended exact T2TF algorithm is straightforward.

This implies sensor networks where a – in general unreliable – communication is employed, but the sensors can regularly transmit or store their locally utilized models as long as it is possible to guarantee that the globalized likelihood is known when the bias is determined. Optimizations to bundle model information from different sensors are conceivable that sum up the measurement matrix combinations $\left(\mathbf{H}_k^j\right)^T \left(\mathbf{C}_k^{v_j}\right)^{-1} \mathbf{H}_k^j$ of a subset of the sensors as only the globalized likelihood (as the sum of the local measurement information) is needed to obtain the bias. In order to further reduce the communication effort, the sensors can utilize a standard measurement model as long as the measurement noise is overestimated and adapt the model only when the difference between real and assumed noise is to big or the noise would have been underestimated. This allows the sensors to transmit measurement models only in (rare) cases when the assumed model differs considerably from the assumed one.

An application of the proposed algorithm that suggests a possibility of handling the actually utilized measurement models is given in the next example. It should be mentioned that in this example the storage of data is minimized instead of the data transfer. This, however, is an equivalent problem in this context.

Example VI.1 In this final example, we do not only consider a limited sensor measurement area and take the sensor failure rate into account, but also allow the measurement quality to be distance-dependent. This scenario is a challenging extension to example III.2 and allows us to demonstrate the application of the proposed algorithm in a more complex environment. We discretize the space of measurement noises into 15 matrices and assume the true measurement noise to be smaller than the assumed one. Therefore, the assumed noise is obtained by $d \cdot \mathbf{C}_k^{v_j}$, where d is a factor depending on the distance of the sensor position $\underline{p}^{v_i}$ and the object position $\underline{p}_k^x$ that is obtained by $d = \mathrm{ceil}\left(\left|\underline{\mathrm{p}}^{\mathrm{v_i}} - \underline{\mathrm{p}}_{\mathrm{k}}^{\mathrm{x}}\right|_2\right)$. When d is larger than 15, the sensor does not obtain a measurement and thus, sets $(\mathbf{H}_k)^T \left(\mathbf{C}_k^v\right)^{-1} \mathbf{H}_k = \mathbf{0}$ and $d = 0$.

The 4 bit for the distance factor are stored in every time step to allow a reconstruction of the utilized measurement model later on. When we assume that it is sufficient to store an estimate every 10th time step and the state variables can be stored in 32 bits each, the entire storage needed for 100 time steps is obtained by

$$\frac{100}{10} \cdot 4 \cdot 32 \text{ bit} + 100 \cdot 4 \text{ bit} = 1680 \text{ bit} = 210 \text{ byte}\,. \qquad (41)$$

Thus, the percentage for storing discretized measurement information is approximately 24% ($\sim 100 \cdot \frac{400 \text{ bit}}{1680 \text{ bit}}$) in this scenario.

For the sake of simplicity, we determine the assumed globalized likelihood by only taking the sensor failure rate into account and neglect the number of sensors that simultaneously detect the object as well as the distance dependent measurement matrices. Although considering these facts would lower the bias and therefore the RMSE of the unbiased exact T2TF result, we will not investigate this here since this topic is out of scope of this paper and the effects are negligible.

A comparison of the unbiased T2TF algorithm with the BLUE estimator is given in Figure 4. The quality of the measurements is significantly worse than in the other scenarios since the measurement noise is multiplied by a factor between 1 and 15 and

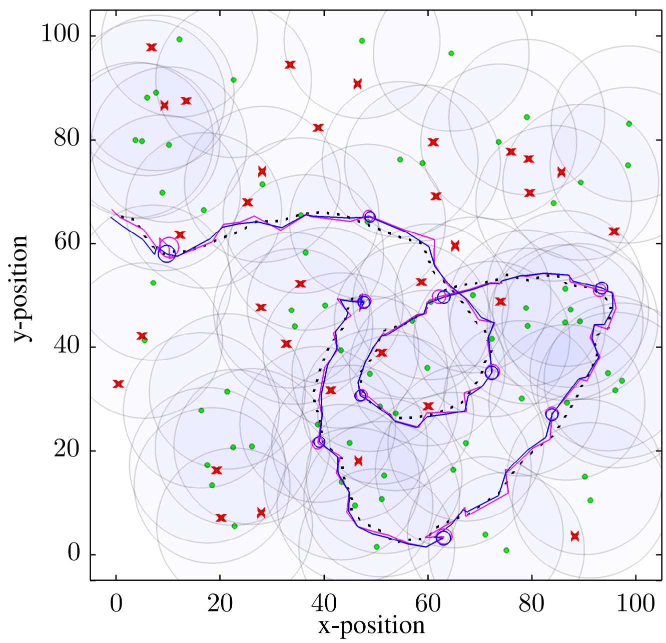

Figure 4. The notation is the same as in the previous figures. The estimate of the unbiased exact T2TF algorithm is given as the magenta coloured path. The true path is dotted in black and the BLUE solution is blue again.

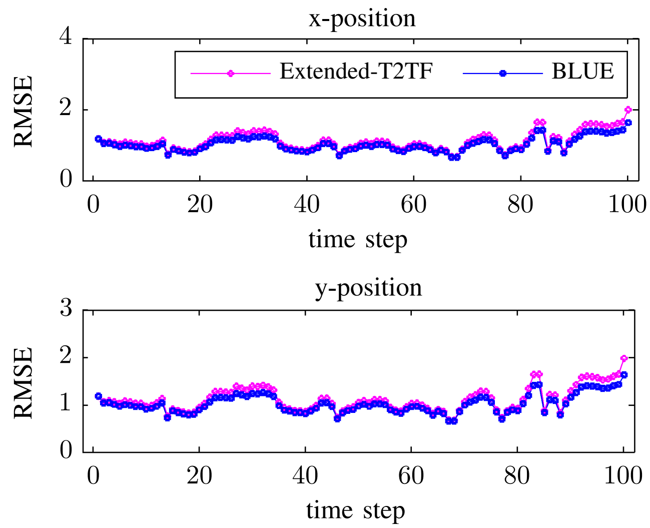

Figure 5. The RMSE of the unbiased exact T2TF algorithm compared to the optimal results.

thus, the BLUE solution as well as the unbiased T2TF algorithm is no longer exactly matching the true path.

As it is possible to calculate the RMSE of the BLUE solution as well as of the unbiased exact T2TF algorithm in closed form, we compare these values at different time steps – instead of the noise depending instances of one run – in Figure 5. By taking the ellipses that mark every 10th time step in Figure 4 into account, it can be seen that the RMSE of both procedures is increasing in areas with low sensor coverage. The difference of the RMSE is remarkably low, which means that the unbiased T2TF algorithm nearly achieves the best possible performance of central data fusion in this scenario, even if it is working distributed under unreliable conditions.

As estimates from sensors that are not available at a calculation node can be predicted from the last time step to the current time step, it is always possible to obtain an estimate of the current state by adding zero matrices for the unknown estimates to the real globalized likelihood of each time step – that means, by implicitly assuming that the sensors have not obtained any measurements since the last information exchange.

In particular, this allows each sensor to obtain an unbiased estimate, even if this sensor only holds estimates of some of his neighbours or in the extreme case, only holds its own estimate. This, however, enables the local sensors to apply nonlinear algorithms such as the Extended Kalman filter or the Unscented Kalman filter. Therefore, the proposed extension to the exact T2TF algorithm provides flexible and precise estimates that are exact, when the globalized likelihood is estimated correctly.

In order to emphasize this, it is not necessary to estimate the local measurement matrix information correctly. As long as the globalized likelihood (as the sum of the measurement matrix information) matches the assumed one, the estimate is exact. For small differences between the real and assumed globalized likelihood, the globally fused estimate is almost exact and otherwise, the estimate is still consistent and unbiased.

Finally, it is worth mentioning that the proposed extension to the exact T2TF algorithm always provides exact results when the process noise is negligible. This is easily proven by setting $\mathbf{C}_k^w = \mathbf{0}$ in (34) and comparing the exact covariance matrices of the subsequent filter step with the unbiased ones.

VII. Conclusion

Implementing Kalman filtering algorithms in a distributed manner is still challenging. In [12], [14], it was shown that prediction and filtering can be performed locally and the local estimates can be fused to a global one that is equal to the result of a centralized Kalman filter. However, the global fusion result can only be computed when all local estimates are available and every node has been able to determine the actual globalized likelihood function in every filtering step, i.e., was aware of every other node and its models. If the underlying assumptions on the used sensor models are not fulfilled all the time, not even a suboptimal fusion result is attained, but a biased result, since the local estimates of this approach do not represent themselves valid and consistent estimates. As illustrated in Sec. IV, a bias is even introduced when one attempts to formulate cautious and conservative assumptions on the network's behavior, e.g., failure rates.

Therefore, we have shown in Sec. V how to compute a correction matrix from the utilized measuring characteristics that removes the bias from the fusion result. It is open to the user to utilize measurement models that allow an easy discretization for efficient storage and transmission as long as the sum of these models is a conservative bound of the real globalized likelihood. In particular, in scenarios where only the rate of sensor failures is of interest, the bias correction matrix can be obtained without additional data transmission needed. Additionally, the proposed approach now allows to use nonlinear filter methods, such as the Extended or Unscented Kalman filter, within the T2TF framework of [12], [14].

The corrected fusion result is an almost optimal estimate of the state and the corresponding covariance matrix embodies the actual mean-squared-error matrix. When the assumed globalized likelihood equals the real one, the results are moreover equivalent to those of the central fusion, while the precision degrades proportional to the deviation between assumed and real models.

Future research will focus on the derivation of precise quality guarantees. For example, the authors observed the mean-squared-error matrix of the proposed approach to be always lower than the assumed covariance matrix when the real globalized uncertainty is lower than the assumed one, independent of the deviation between the assumed and the real globalized likelihood.

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