Association-Free Multilateration Based on Times of Arrival

Daniel Frisch and Uwe D. Hanebeck

***Abstract*— Multilateration systems reconstruct the location of a target that transmits electromagnetic or acoustic signals. The employed measurements for localization are the times of arrival (TOAs) of the transmitted signal, measured by a number of spatially distributed receivers at known positions. We present a novel multilateration algorithm to localize multiple targets that transmit indistinguishable signals at unknown times. That is, each receiver measures merely a set of TOAs with no association to the targets. Our method does not need any prior information. Therefore, it can provide uncorrelated, static measurements to be introduced into a separate tracker subsequently, or an initialization routine for multi target trackers.**

I. Introduction

Context: Electromagnetic and acoustic signals have various properties that carry information and, when received at multiple sites, can be exploited to obtain the location of the signal transmitter (target). In multilateration (MLAT), the time of arrival (TOA) is used for target localization. A typical example is found in aircraft surveillance, where transmitters on every aircraft emit so-called Mode A/C signals. Since that standard only allows for 4096 addresses, there are often multiple aircraft with the same address in one airspace. Mode A/C transmissions are triggered by interrogator stations, therefore it is common that many targets transmit shortly after each other. As a consequence, receivers often cannot separate targets just by temporal distance of the TOAs. Other application scenarios of the presented method include localizing objects emitting indistinguishable acoustic events.

State of Art: Signal properties that carry information about the location of its emitter are for example received signal strength (RSS) [2] and angle of arrival (AOA), also known as bearings measurements [3], [4]. The most accurate location estimates can typically be obtained with TOA measurements [5], [6], or differences of TOA [7], [8], [9] for precisely time-synchronized receiver networks. This is generally referred to as MLAT. In secondary surveillance radar (SSR), signal transmissions are actively triggered by an interrogator, and the duration between sending the interrogation and reception of the target's response gives an additional indication of the target's distance [10]. However, if the time lag the target needs to process the interrogation is not accurately known, this information should be used with care. Two common SSR variants are Mode S and Mode A/C.

This work was funded by our collaboration partner Frequentis Comsoft GmbH. The methods described herein are protected by a patent application from Frequentis Comsoft GmbH and Karlsruhe Institute of Technology (KIT) [1]. Daniel Frisch and Uwe D. Hanebeck are with the Intelligent Sensor-Actuator-Systems Laboratory (ISAS), Institute for Anthropomatics and Robotics, Karlsruhe Institute of Technology (KIT), Germany. E-mails: daniel.frisch@posteo.de, uwe.hanebeck@kit.edu.

For multi target tracking, the main challenge is that the association between measurements and targets is unknown. If the associations were known because transmissions contain individual codes, the problem can simply be reduced to single target tracking, separately for every target. Multi target tracking can be treated with the Joint Probabilistic Data Association Filter [11], but this requires enumerating all possible association hypotheses. Other methods like the probability hypothesis density filter avoid the measurement associations in their formulation [12], [13]. Further interesting possibilities are symmetric and association-invariant transformations like the SME Filter [14], the Kernel-SME Filter [15], [16], or methods based on association-invariant set distance measures [17], [18]. Most methods however have been proposed for linear measurement models or measurements with the full dimensionality only, and cannot easily be applied to nonlinear subspace measurements like TOA for emitter localization.

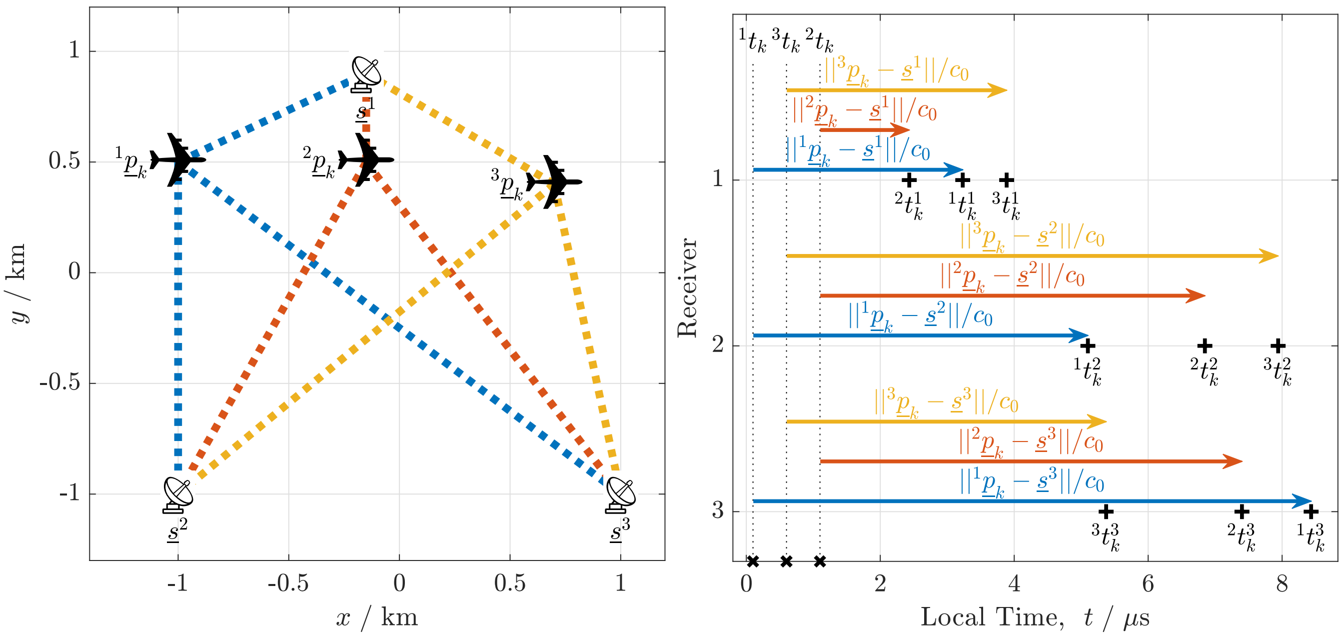
Contribution: In this paper, we will consider a multi target localization setup with realistic TOA measurements that implicate a nonlinear measurement model. Our approach can be used to instantly locate a number of objects transmitting identical messages without any prior knowledge.

II. Problem Formulation

We consider the localization of multiple targets based on noisy TOA measurements from a receiver network, where the target-to-measurement associations are unknown. Specifically, we make the following assumptions:

- A1 The number of targets $\mathcal{P}$ is known.
- A2 There can be missed detections or false measurements (but not both) at one receiver.
- A3 TOA measurements are obtained from synchronized, passive receivers at known locations.

We consider a set of $\mathcal{P}$ stationary or moving targets during a number of dense transmission events $k \in \{1, 2, \dots, K\}$. With dense transmission events, we indicate situations where multiple targets transmit quickly in a row, such that the messages are received in different orders on different receivers. Target positions are denoted by ${}^{j}\underline{p}_k \in \mathbb{R}^{\mathcal{D}}$, where $j \in \{1, 2, \dots, \mathcal{P}\}$ is the target index, and dimension $\mathcal{D}$ is typically two or three. Every target emits electromagnetic or acoustic messages at time steps ${}^{j}t_k$ which propagate with uniform propagation speed c_0. These target transmission times (TTTs) do not need to be known beforehand. Therefore, the target clock does not need to be synchronized with any other clock, and transmissions can be equi-spaced in time or not. Target transmissions also do not need to be triggered by an interrogation signal. It merely must be provided that each target does transmit from time to time, because its location



(a) (b)

Fig. 1: Geometry plot (a) shows the position of three aircraft (targets) $^j\underline{p}_k,\ j \in \{1,2,3\}$ at the time instances ^jt_k. Transmitted messages propagate with velocity c_0 into all directions and are detected by the receivers $\underline{s}^i,\ i \in \{1,2,3\}$. In (b) you can study the same scenario on a time axis. Transmission of target one $^1\underline{p}_k$ takes place at $^1t_k = 0.1\,\mu\text{s}$. From there, the signal travels through space (blue arrows) and reaches the individual receivers at times $^1t_k^i$ (indicated with fat plus signs). Targets two and three transmit at $^2t_k = 1.1\,\mu\text{s}$ and $^3t_k = 0.6\,\mu\text{s}$, respectively, and these transmissions propagate likewise (red and yellow arrows). Because targets transmit in quick succession, the order of TOA measurements from the three targets varies among the receivers. Therefore, a dense transmission event takes place, indicated with k.

can be measured only for the moments of transmission. Also, the signal must be shaped in a way that a specific TOA can be extracted on all receivers referring to the same event.

Measurements are obtained by $\mathcal{S}$ receivers stationed at known locations $\underline{s}^i \in \mathbb{R}^{\mathcal{D}},\ i \in \{1,2,\ldots,\mathcal{S}\}$. These receivers record the TOAs $^jt_k^i$ of the transmissions. Thereby the receivers are synchronized among each other, but they need not to be synchronized with any target. The basic equation that connects aforementioned variables is

$$^jt_k^i = {}^jt_k + \frac{\left\|{}^j\underline{p}_k - \underline{s}^i\right\|}{c_0}\ , \tag{1}$$

see also Fig. 1. Our goal is to determine the target positions $^j\underline{p}_k$ of all $\mathcal{P}$ targets based on the $(\mathcal{P}\cdot\mathcal{S})$ TOAs $^jt_k^i$ measured in the $\mathcal{S}$ receivers.

Traditional MLAT methods usually employ differences of TOA measurements from different receivers, i.e.,

$$^jt_k^{i_2} - {}^jt_k^{i_1} = \cancel{{}^jt_k} + \frac{\left\|{}^j\underline{p}_k - \underline{s}^{i_2}\right\|}{c_0} - \cancel{{}^jt_k} - \frac{\left\|{}^j\underline{p}_k - \underline{s}^{i_1}\right\|}{c_0}\ .$$

That way, the unknown TTT ^jt_k is gone from the measurement equation and does not need to be estimated. Of course, this works only if the two transmissions come from the same target j. As we do not know which pairs of TOAs come from the same target, we do not know which pairs to use. Obviously, already taking differences of TOAs increases the number of possible wrong combinations and does *not* seem to be a sensible approach here. Therefore, we decide to go back to the raw TOA measurements and work directly with these. Consequently, our target state $^j\underline{x}_k$ will consist of TTT ^jt_k and target positions $^j\underline{p}_k$

$$^j\underline{x}_k = \begin{bmatrix} {}^j\underline{p}_k \\ {}^jt_k \end{bmatrix}\ .$$

First we define a notation that bundles the states $^j\underline{x}_k$ of all targets $j \in \{1,2,\ldots,\mathcal{P}\}$ into a combined target state $\mathbf{X}_k$, and also the $\mathcal{P}$ TOA measurements $^jt_k^i$ of one receiver i into the measurement vector $\underline{t}_k^i$

$$\mathbf{X}_k = \begin{bmatrix} {}^1\underline{x}_k \\ {}^2\underline{x}_k \\ \vdots \\ {}^{\mathcal{P}}\underline{x}_k \end{bmatrix}\ ,\quad \underline{h}^i(\mathbf{X}_k) = \begin{bmatrix} h^i({}^1\underline{x}_k) \\ h^i({}^2\underline{x}_k) \\ \vdots \\ h^i({}^{\mathcal{P}}\underline{x}_k) \end{bmatrix}\ ,\quad \underline{t}_k^i = \begin{bmatrix} {}^1t_k^i \\ {}^2t_k^i \\ \vdots \\ {}^{\mathcal{P}}t_k^i \end{bmatrix}\ ,$$

where $h^i({}^j\underline{x}_k)$ is the measurement equation (1) for receiver i at location $\underline{s}^i$,

$$h^i({}^j\underline{x}_k) = {}^jt_k + \frac{\left\|{}^j\underline{p}_k - \underline{s}^i\right\|}{c_0}\ . \tag{2}$$

Summarizing, we have a nonlinear measurement equation system for each receiver i, namely

$$\underline{t}_k^i = \underline{h}^i(\mathbf{X}_k) \ . \tag{3}$$

The *real* measurements ${}^{\pi_k^i(j)}\hat{t}_k^i$ are additionally distorted with additive noise ${}^j v_k^i$ caused by e.g. a low signal-to-noise ratio. But most notably, ${}^{\pi_k^i(j)}\hat{t}_k^i$ can be correctly compared with the hypothesized measurements (3) only if the true measurement-to-target association function $\pi_k^i(j)$ has been found

$$\underbrace{\begin{bmatrix} {}^{\pi_k^i(1)}\hat{t}_k^i \\ {}^{\pi_k^i(2)}\hat{t}_k^i \\ \vdots \\ {}^{\pi_k^i(\mathcal{P})}\hat{t}_k^i \end{bmatrix}}_{{}^{\pi_k^i}\hat{\underline{t}}_k^i} = \underbrace{\begin{bmatrix} h^i({}^1\underline{x}_k) \\ h^i({}^2\underline{x}_k) \\ \vdots \\ h^i({}^{\mathcal{P}}\underline{x}_k) \end{bmatrix}}_{\underline{h}^i(\mathbf{X}_k)} + \underbrace{\begin{bmatrix} {}^1 v_k^i \\ {}^2 v_k^i \\ \vdots \\ {}^{\mathcal{P}} v_k^i \end{bmatrix}}_{\underline{v}_k^i} \ .$$

Combined for all $\mathcal{S}$ receivers, we obtain the equation system

$$\underbrace{\begin{bmatrix} {}^{\pi_k^i}\hat{\underline{t}}_k^1 \\ {}^{\pi_k^i}\hat{\underline{t}}_k^2 \\ \vdots \\ {}^{\pi_k^i}\hat{\underline{t}}_k^{\mathcal{S}} \end{bmatrix}}_{{}^{\pi_k}\hat{\mathbf{T}}_k} = \underbrace{\begin{bmatrix} \underline{h}^1(\mathbf{X}_k) \\ \underline{h}^2(\mathbf{X}_k) \\ \vdots \\ \underline{h}^{\mathcal{S}}(\mathbf{X}_k) \end{bmatrix}}_{\mathbf{H}(\mathbf{X}_k)} + \underbrace{\begin{bmatrix} \underline{v}_k^1 \\ \underline{v}_k^2 \\ \vdots \\ \underline{v}_k^{\mathcal{S}} \end{bmatrix}}_{\mathbf{V}_k} \ ,$$

or in short form

$${}^{\pi_k}\hat{\mathbf{T}}_k = \mathbf{H}(\mathbf{X}_k) + \mathbf{V}_k \ .$$

This equation system contains all information from $\mathcal{S}$ receivers with $\mathcal{P}$ TOA measurements each. As no prior information is assumed, we want to obtain the maximum likelihood estimate. In order to obtain the likelihood, we first define the conditional density

$$f(\hat{\mathbf{T}}_k \,|\, \underline{\pi}_k, \mathbf{X}_k, \mathbf{V}_k) = \delta({}^{\pi_k}\hat{\mathbf{T}}_k - \mathbf{H}(\mathbf{X}_k) - \mathbf{V}_k) \ ,$$

and by the law of conditional densities, the joint density

$$f(\hat{\mathbf{T}}_k, \mathbf{V}_k \,|\, \underline{\pi}_k, \mathbf{X}_k) = \delta({}^{\pi_k}\hat{\mathbf{T}}_k - \mathbf{H}(\mathbf{X}_k) - \mathbf{V}_k) \cdot f_k^{\mathbf{V}}(\mathbf{V}_k) \ ,$$

where $f_k^{\mathbf{V}}(\mathbf{V}_k)$ is the joint density of all measurement noises $\mathbf{V}_k$. Now we marginalize over $\mathbf{V}_k$, this yields the likelihood function Λ of the unknowns $\underline{\pi}_k, \mathbf{X}_k$

$$\begin{aligned} \Lambda(\underline{\pi}_k, \mathbf{X}_k) &= f_k^{\mathrm{L}}(\hat{\mathbf{T}}_k \,|\, \underline{\pi}_k, \mathbf{X}_k) \\ &= f_k^{\mathbf{V}}({}^{\pi_k}\hat{\mathbf{T}}_k - \mathbf{H}(\mathbf{X}_k)) \ . \end{aligned}$$

The maximum likelihood estimate of measurement associations and combined target state can be obtained by maximizing the likelihood function

$$\left(\hat{\underline{\pi}}_k^{\mathrm{ML}}, \hat{\mathbf{X}}_k^{\mathrm{ML}}\right) = \arg\max_{\underline{\pi}_k, \mathbf{X}_k} \left\{\Lambda(\underline{\pi}_k, \mathbf{X}_k)\right\} \ .$$

Assuming that $f_k^{\mathbf{V}}$ is a multivariate Gaussian density with a diagonal covariance matrix, i.e.,

$$\mathrm{Cov}\left\{{}^{j_1}v_{k_1}^{i_1}, {}^{j_2}v_{k_2}^{i_2}\right\} = \left({}^{j_1}\sigma_{k_1}^{i_1}\right)^2 \delta_{j_1,j_2}\delta_{k_1,k_2}\delta_{l_1,l_2} \ ,$$

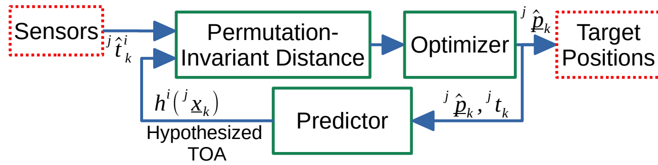


Fig. 2: Block diagram of the proposed multi target localization algorithm.

the likelihood maximization problem can be formulated somewhat more intuitively as distance minimization problem

$$\left(\hat{\underline{\pi}}_k^{\mathrm{ML}}, \hat{\mathbf{X}}_k^{\mathrm{ML}}\right) = \arg\min_{\underline{\pi}_k, \mathbf{X}_k} \left\{ \sum_{i=1}^{\mathcal{S}} \sum_{j=1}^{\mathcal{P}} \left(\frac{{}^{\pi_k^i(j)}\hat{t}_k^i - h^i({}^j\underline{x}_k)}{{}^j\sigma_k^i} \right)^2 \right\} .$$

Either way this turns out to be a mixed combinatorial and continuous optimization problem. To convey an idea of the complexity, we will indicate the number of possibilities for the combinatorial part. For one specific receiver i, the association function $\pi_k^i(j)$, $j \in \{1, \ldots, \mathcal{P}\}$, can yield $\mathcal{P}!$ different permutations. The overall number of possibilities to be considered for all $\mathcal{S}$ receivers is $(\mathcal{P}!)^{\mathcal{S}}$, which may be reduced to $(\mathcal{P}!)\cdot\mathcal{S}$ if associations can be optimized for each receiver independently.

III. Key Idea

Instead of going through all possible associations, we propose a mathematical *composition* of an outer continuous optimization loop and an inner combinatorial optimization, see Fig. 2. The outer loop always chooses new target positions and TTT. The inner combinatorial part then calculates the maximum likelihood that is possible for these values, i.e., how well they fit to the measurements. Based on that information, the outer optimization loop can proceed to the next iteration step and try other target positions until the likelihood is maximized globally. We will now describe in detail how this alternating optimization procedure can be achieved efficiently.

IV. Combinatorial Optimization Procedure

In the view of the inner combinatorial optimization loop, we are always given a fixed combined target state $\hat{\mathbf{X}}_{k,l}$ (target positions and TTT at iteration l) that is suggested by the outer loop. We then have to find the association permutation $\hat{\underline{\pi}}_{k,l}$ that aligns both sets of TOAs such that the likelihood is maximized

$$\begin{aligned} \hat{\underline{\pi}}_{k,l} &= \arg\max_{\underline{\pi}_k} \left\{\Lambda(\underline{\pi}_k, \hat{\mathbf{X}}_{k,l})\right\} \\ &= \arg\min_{\underline{\pi}_k} \left\{ \sum_{i=1}^{\mathcal{S}} \sum_{j=1}^{\mathcal{P}} \left(\frac{{}^{\pi_k^i(j)}\hat{t}_k^i - h^i({}^j\hat{\underline{x}}_{k,l})}{{}^j\sigma_k^i} \right)^2 \right\} \ . \end{aligned}$$

In fact, we may not even be interested in the actual association but only in the maximum likelihood $\hat{\Lambda}$ or the minimum distance $\hat{D}$ to be obtained over all possible permutations $\underline{\pi}_k$ for the given $\hat{\mathbf{X}}_{k,l}$

$$\hat{\Lambda}(\hat{\mathbf{X}}_{k,l}) = \max_{\underline{\pi}_k} \left\{\Lambda(\underline{\pi}_k, \hat{\mathbf{X}}_{k,l})\right\} \ ,$$

$$\hat{D}(\hat{\mathbf{X}}_{k,l}) = \min_{\underline{\pi}_k} \left\{ \sum_{i=1}^{\mathcal{S}} \sum_{j=1}^{\mathcal{P}} \left(\frac{{}^{\pi_k^i(j)}\hat{t}_k^i - h^i({}^j\hat{\underline{x}}_{k,l})}{{}^j\sigma_k^i} \right)^2 \right\} \quad .$$

Fortunately, changing only the measurement association π_k^i at one receiver while keeping the combined target state fixed does not influence the quadratic difference terms contributed by the other receivers in $\hat{D}$. Therefore, association permutations can be optimized for each receiver individually

$$\hat{D}(\hat{\mathbf{X}}_{k,l}) = \sum_{i=1}^{\mathcal{S}} \left(\min_{\pi_k^i} \left\{ \sum_{j=1}^{\mathcal{P}} \left(\frac{{}^{\pi_k^i(j)}\hat{t}_k^i - h^i({}^j\hat{\underline{x}}_{k,l})}{{}^j\sigma_k^i} \right)^2 \right\} \right) \quad .$$

Now what happens exactly for an individual receiver intuitively? We want to calculate the function

$$\hat{D}^i(\hat{\mathbf{X}}_{k,l}) = \min_{\pi_k^i} \left\{ \sum_{j=1}^{\mathcal{P}} \left(\frac{{}^{\pi_k^i(j)}\hat{t}_k^i - h^i({}^j\hat{\underline{x}}_{k,l})}{{}^j\sigma_k^i} \right)^2 \right\} \quad . \quad (4)$$

It consists of simply calculating the expected TOA measurements based on ${}^j\hat{\underline{x}}_{k,l}$ using (2). As a result, we have two sets of TOAs for each receiver: the hypothesized TOAs $h^i({}^j\hat{\underline{x}}_{k,l})$ based on the suggested state ${}^j\hat{\underline{x}}_{k,l}$, and the TOAs ${}^j\hat{t}_k^i$ that were actually measured by the receiver. We want to find a permutation π_j^k to reorder the latter, such that the sum of the weighted quadratic differences between associated TOA is minimized. This is an optimal transport problem, or more specifically, a discrete Monge problem, which is an assignment problem. The resulting $\hat{D}^i$ (4) is a squared Wasserstein distance [19].

A. Sorting the TOAs

TOA measurements are of course one-dimensional and thus have a natural order that can be easily obtained by sorting. Now if two sets (A, B) of TOAs with the same cardinality are ordered from earliest to latest, and the weighting of the individual squared differences is uniform, then obviously the smallest overall distance can be obtained by associating elements according to their indices in the sorted lists, i.e., associating the earliest TOA of set A with the earliest TOA of set B, and so on. In other words, if there are no missed detections and no clutter, and the measurement noise variances ${}^j\sigma_k^i$ for all targets j are the same at receiver i, then the contribution $\hat{D}^i(\hat{\mathbf{X}}_{k,l})$ of that receiver to the overall distance $\hat{D}(\hat{\mathbf{X}}_{k,l})$ can be very efficiently obtained by sorting.

B. Try Few Possibilities

In the case of $\mathcal{F}$ *false* measurements in a clutter environment, the cardinality of the two sets of TOAs is not equal anymore: the set of measured TOAs consists of $\mathcal{P} + \mathcal{F}$ measurements. Both sets may still be sorted, but $\mathcal{F}$ elements from the set with more elements are not assigned. There are

$$\binom{\mathcal{P} + \mathcal{F}}{\mathcal{F}} = \frac{(\mathcal{P} + \mathcal{F})!}{\mathcal{P}! \cdot \mathcal{F}!}$$

possibilities to choose the $\mathcal{F}$ elements to exclude. In the case of one single false measurement ($\mathcal{F}=1$), these are only $(\mathcal{P} + 1)$ possibilities, or $\frac{1}{2}(\mathcal{P} + 2)(\mathcal{P} + 1)$ for ($\mathcal{F}=2$). When there are instead $\mathcal{M}$ *missed* measurements, we have

$$\binom{\mathcal{P}}{\mathcal{M}} = \frac{\mathcal{P}!}{(\mathcal{P} - \mathcal{M})! \cdot \mathcal{M}!}$$

possibilities to exclude $\mathcal{M}$ of the $\mathcal{P}$ TOAs from association. Trying out all possibilities and applying Sec. IV-A each time can be feasible for small numbers of $\mathcal{F}$ or $\mathcal{M}$.

C. Minimum of a List

If only *one* measurement has been detected at the considered receiver, i.e., $\mathcal{P} - \mathcal{M} = 1$, then simply the nearest TOA is assigned to it, so the necessary operation is taking the minimum of a list. Vice versa, the same holds if there are multiple clutter measurements but only one target ($\mathcal{P}=1$).

D. Linear Programming

If the number of false $\mathcal{F}$ or missed $\mathcal{M}$ measurements is too high to apply Sec. IV-B, or if TOAs from multiple targets have individual variances, i.e., the TOA differences in (4) have individual weightings ${}^j\sigma_k^i$, then the association problem should be solved with a dedicated linear assignment algorithm like the Hungarian algorithm [20] or Munkres assignment algorithm [21]. It was originally proposed for square cost matrices but can be extended to rectangular matrices [22]. These algorithms always find the optimal assignment in polynomial time. An alternative is the Auction algorithm [23], which may obtain a suboptimal solution, but is faster and can be solved in real time even for larger sets of objects [24], [25].

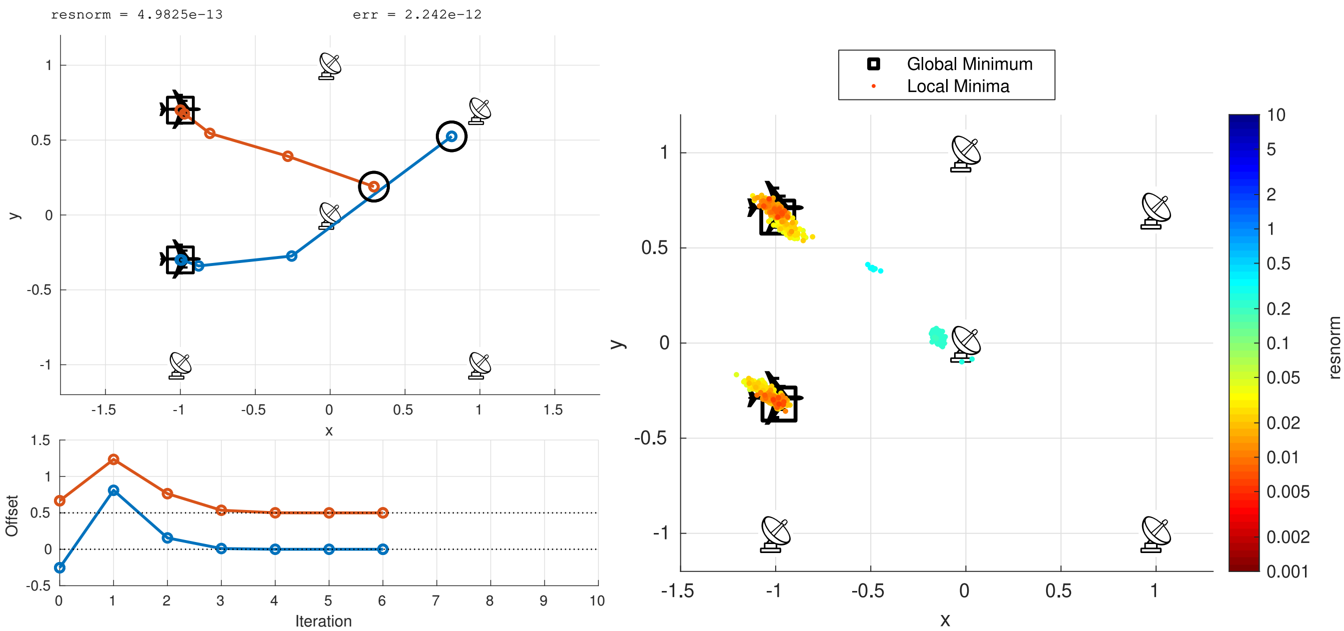
V. Continuous Optimization Procedure

The outer optimization loop performs standard continuous optimization, where the objective function $\hat{D}$ is a sum of squared Wasserstein distances $\hat{D}^i$ (4). Therefore, it is a nonconvex nonlinear least-squares problem. Such nonlinear least-squares problems can be solved with the Levenberg-Marquardt algorithm [26] or a trust-region algorithm [27]. The solver must be given a suitable initial value, for example randomly chosen target positions over the area covered by receivers, if no prior knowledge about target positions is available. As the problem is not convex, the solver tends to get trapped in local minima. To be sure to find the global optimum without using prior knowledge, multiple runs with randomly chosen initial values should be performed. The set of target positions found by the optimization run that yielded the smallest residual norm (objective function value) after convergence is selected then. For better convergence and performance, the least squares solver should also be provided with analytical gradients in addition to the squared Wasserstein distances $\hat{D}^i$ (4).

VI. Evaluation

A. Synthetic Data

In order to evaluate the basic applicability of our algorithm and implementation, a two-dimensional synthetic setup with five receivers and two targets was compiled, see Fig. 3 (a).



(a)

(b)

Fig. 3: (a) Visualization of the iterations of the continuous optimization routine. True positions of aircraft and receivers are indicated by pictograms. The randomly chosen start points are shown as open circles, and the final result of the optimization as open squares. In between, the intermediate iteration steps are shown as red and blue dots and curves. The lower part of the figure visualizes the optimization progress of the TTT. The true TTT of the upper target ${}^1\underline{p}_k = [-1, 0.7]^\top$ is ${}^1t_k = 0.5$, and that of the lower target ${}^2\underline{p}_k = [-1, -0.3]^\top$ is ${}^2t_k = 0$. Propagation velocity c_0 has been set to one here, so everything can be done without units. (b) Target positions resulting from several optimization runs with different, randomly chosen initial points (to circumvent local minima) and additive measurement noise. Points are colored by the final residual norm. Evidently, the turquoise points represent local minima. On the other hand, the result with the smallest residual norm, i.e., the global optimum, is additionally indicated by an open square and agrees well with the true target positions (aircraft pictograms). See the supplemental video for an animated version of this figure.

Neither measurement noise nor detection misses were included here. Initial points for the two targets were randomly chosen with an equal distribution over the entire plotted area, and initial TTT were chosen equally distributed in a window of one around the respective true value.

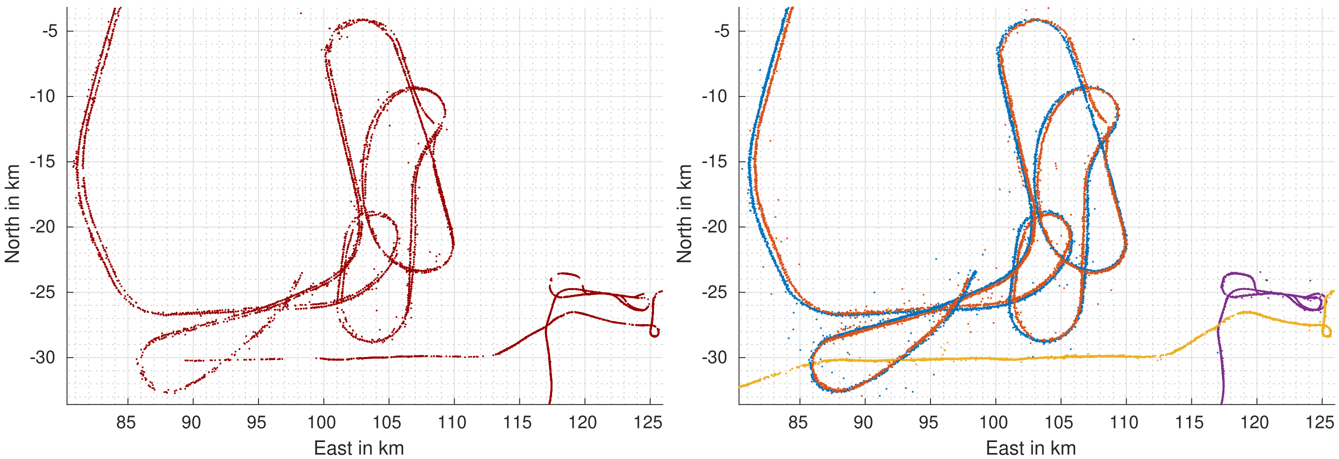
In a second step, simulations were performed where Gaussian noise with ${}^j\sigma_k^k = 0.02$ was added to the measurements, but no detection misses. The resulting target positions of 500 runs with randomly chosen start points are shown in Fig. 3 (b). Results are also colored by their residual norm, i.e., the final value of $\sqrt{\hat{D}}$. The supplemental video shows a similar plot for moving targets. Calculation time for the 500 runs was about $1.1\,\mathrm{s}$ without multithreading.

B. Real Data

Together with Frequentis Comsoft GmbH we analyzed real data from a $20\,\mathrm{min}$ national air traffic control data set based on the Mode A/C radio frequency communication protocol. Overall, it provided $440\,532$ TOA events with 4090 different Mode C target identification codes. All targets actually had individual codes and the targets thus could also be localized without association-free MLAT, but this information was used for ground truth only. We focused on four specific targets with the codes 2415, 2416, 4442, 5403. This restricted the data set to $157\,585$ TOAs from 14 receivers covering roughly $30\,000\,\mathrm{km}^2$, where the four considered targets stayed within an area of about $3500\,\mathrm{km}^2$. Frequentis Comsoft Quadrant interrogators always transmit in a specific A-C-A-A pattern of four Mode A and Mode C requests, in order to be able to recognize Mode A/C messages that were triggered by own interrogations. Furthermore, the four answers in combination provide a more accurate single TOA measurement. The data set has been specifically chosen for testing our association-free multi target localization method. Targets perform quite complex maneuvers simultaneously, two of them in close formation. On the other hand, the environment is chosen such that there are no other major difficulties like reflections present. We can thus assume that measurements are free of clutter, but with missed detections.

C. Procedure

The Mode C transmissions contain pressure altitude, but this information was not used – only the TOA measurements of the transmissions together with the known receiver positions. Mode A transmissions contain a 12-bit code to identify the targets and all considered targets had unique



(a) (b)

Fig. 4: MLAT results for four targets based on *real* data from Frequentis Comsoft Quadrant air traffic surveillance. The problem geometry is 3D, but altitude is not shown here. (a) Received TOA events were treated as association-free (ambiguous) measurements and target localization was performed with the proposed association-free MLAT algorithm. (b) Corresponding ground truth. Codes of the Mode A messages were used to identify targets and perform separate single target localizations. Each color indicates an individual Mode A code, i.e., a specific target. Supplemental video contains animated version of (a).

IDs, but this information was only used as ground truth in order to evaluate the proposed multi target localization method. The maximum distance between two receivers was $233\,\mathrm{km}$, so pauses of $0.78\,\mathrm{ms}$ and more could be seen as a guaranteed separation between dense transmission events. The A-C-A-A interrogation sequence was emitted in intervals of at least $10\,\mathrm{ms}$, therefore $10\,711$ dense transmission events were clearly separated. According to the Mode A codes from ground truth, 9873 of the dense transmission events actually contained messages from more than one target. In order to be able to apply our method without further modifications, we determined the number of targets $\mathcal{P}$ from ground truth for every dense transmission event. For the MLAT algorithm, the actual number of targets behind a set of TOA is however not known in reality. This will be addressed in future works.

D. Results

The multi target localization performance on real data achieved by the proposed method can be seen in Fig. 4 (a). The plot shows only point estimates based on six or more TOA measurements per point. In the supplemental video, the same MLAT results are shown over time, and the evolution of the four tracks can be followed more intuitively there.

VII. Conclusion

We propose an efficient algorithm for MLAT that does not require the association between receiving events and tracked targets. Evaluations with synthetic and real data demonstrated good localization accuracy with ambiguous TOA measurements.

Usually, the receiving events comprise scalar (TOA) measurements. For scalar measurements, the proposed method is especially fast and efficient as finding optimal permutation just entails a sorting procedure. Furthermore, the optimal permutation can be obtained for each receiver independently, which reduces overall complexity and facilitates parallel execution.

VIII. Future Work

So far, the problem was assumed to be static, i.e., we obtain single-shot point estimates that are independent from each other. We plan to expand this method to a target tracking system. Therefore, we introduce prior knowledge and combine suitable dynamic motion models with the proposed measurement model. This might also vastly reduce computational power if the iterative continuous optimization loop can be replaced with a single Kalman filter step, for example. Batch processing algorithms could be interesting to determine inherent system properties like measurement variances and typical deviations in clock synchronization.

An important extension is the automatic detection of the number of targets behind a set of received TOA events. For example, suppose that 14 receivers detected one TOA event each. It could be that all of them come from one target, or seven from two different targets, respectively. We plan to heuristically narrow down the possible number of targets and exploit hypothesis probabilities based on the chi distribution. Finally, in many applications it is necessary to deal with both clutter and missing measurements at the same time.

IX. Acknowledgment

The authors would like to thank Michael Sharples from Frequentis Comsoft GmbH for selecting and providing data sets with real data from a Quadrant wide area multilateration (WAM) air traffic surveillance system.

References

[1] U. Hanebeck, D. Frisch, B. Noack, and M. Sharples, "Method for Determining the Location of a Plurality of Targets," 2018, European Patent App. EP18214798.3.

[2] Q. Wang, Z. Duan, X.-R. Li, and U. D. Hanebeck, "Convex Combination for Source Localization Using Received Signal Strength Measurements," in *Proceedings of the 21st International Conference on Information Fusion (Fusion 2018)*, Cambridge, United Kingdom, Jul. 2018.

[3] S. Radtke, K. Li, B. Noack, and U. D. Hanebeck, "Comparative Study of Track-to-Track Fusion Methods for Cooperative Tracking with Bearing-only Measurements," in *Proceedings of the 2019 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI 2019)*, Taipei, Republic of China, May 2019.

[4] J. Hörst, F. Sawo, V. Klumpp, U. D. Hanebeck, and D. Fränken, "Extension of the Sliced Gaussian Mixture Filter with Application to Cooperative Passive Target Tracking," in *Proceedings of the 12th International Conference on Information Fusion (Fusion 2009)*, Seattle, Washington, USA, Jul. 2009.

[5] F. Beutler and U. D. Hanebeck, "A Two-Step Approach for Offset and Position Estimation from Pseudo-Ranges Applied to Multilateration Tracking," in *Proceedings of the 13th International Conference on Information Fusion (Fusion 2010)*, Edinburgh, United Kingdom, Jul. 2010.

[6] R. Kaune, C. Steffes, S. Rau, W. Konle, and J. Pagel, "Wide Area Multilateration using ADS-B Transponder Signals," in *2012 15th International Conference on Information Fusion*, July 2012, pp. 727–734.

[7] J. Smith and J. Abel, "Closed-Form Least-Squares Source Location Estimation from Range-Difference Measurements," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 35, no. 12, pp. 1661–1669, 1987.

[8] J. Smith and J. Abel, "The Spherical Interpolation Method of Source Localization," *IEEE Journal of Oceanic Engineering*, vol. 12, no. 1, pp. 246–252, January 1987.

[9] R. Morrissey, J. Ward, N. DiMarzio, S. Jarvis, and D. Moretti, "Passive Acoustic Detection and Localization of Sperm Whales (Physeter Macrocephalus) in the Tongue of the Ocean," *Applied Acoustics*, vol. 67, no. 11, pp. 1091 – 1105, 2006, Detection and Localization of Marine Mammals Using Passive Acoustics.

[10] D. Fränken and A. Hüpper, "Aircraft Positioning with Secondary Surveillance Radar and Time of Arrival Measurements," in *2013 14th International Radar Symposium (IRS)*, vol. 1, June 2013, pp. 355–360.

[11] Y. Bar-Shalom, F. Daum, and J. Huang, "The Probabilistic Data Association Filter," *IEEE Control Systems Magazine*, vol. 29, no. 6, pp. 82–100, 2009.

[12] R. P. Mahler, "Multitarget Bayes Filtering via First-Order Multitarget Moments," *IEEE Transactions on Aerospace and Electronic systems*, vol. 39, no. 4, pp. 1152–1178, 2003.

[20] H. W. Kuhn, "The Hungarian Method for the Assignment Problem," *Naval research logistics quarterly*, vol. 2, no. 1-2, pp. 83–97, 1955.

[13] O. Erdinc, P. Willett, and Y. Bar-Shalom, "Probability Hypothesis Density Filter for Multitarget Multisensor Tracking," in *2005 7th International Conference on Information Fusion*, vol. 1. IEEE, 2005, pp. 8–pp.

[14] E. Kamen, "Multiple Target Tracking Based on Symmetric Measurement Equations," *IEEE Transactions on Automatic Control*, vol. 37, no. 3, pp. 371–374, 1992.

[15] M. Baum, S. Yang, and U. D. Hanebeck, "The Kernel-SME Filter with False and Missing Measurements," in *Proceedings of the 19th International Conference on Information Fusion (Fusion 2016)*, Heidelberg, Germany, Jul. 2016.

[16] M. Baum and U. D. Hanebeck, "The Kernel-SME Filter for Multiple Target Tracking," in *Proceedings of the 16th International Conference on Information Fusion (Fusion 2013)*, Istanbul, Turkey, Jul. 2013.

[17] U. D. Hanebeck, M. Baum, and P. Willett, "Symmetrizing Measurement Equations for Association-free Multi-target Tracking via Point Set Distances," in *SPIE - Signal Processing, Sensor/Information Fusion, and Target Recognition XXVI*, Anaheim, California, USA, Apr. 2017.

[18] U. D. Hanebeck and M. Baum, "Association-Free Direct Filtering of Multi-Target Random Finite Sets with Set Distance Measures," in *Proceedings of the 18th International Conference on Information Fusion (Fusion 2015)*, Washington D.C., USA, Jul. 2015.

[19] G. Peyré and M. Cuturi, "Computational Optimal Transport," *arXiv e-prints*, p. arXiv:1803.00567, Mar 2018.

[21] J. Munkres, "Algorithms for the Assignment and Transportation Problems," *Journal of the society for industrial and applied mathematics*, vol. 5, no. 1, pp. 32–38, 1957.

[22] F. Bourgeois and J.-C. Lassalle, "An Extension of the Munkres Algorithm for the Assignment Problem to Rectangular Matrices," *Communications of the ACM*, vol. 14, no. 12, pp. 802–804, 1971.

[23] D. P. Bertsekas, "A Distributed Algorithm for the Assignment Problem," *Lab. for Information and Decision Systems Working Paper, MIT*, 1979.

[24] F. Pfaff, C. Pieper, G. Maier, B. Noack, H. Kruggel-Emden, R. Gruna, U. D. Hanebeck, S. Wirtz, V. Scherer, T. Längle, and J. Beyerer, "Simulation-Based Evaluation of Predictive Tracking for Sorting Bulk Materials," in *Proceedings of the 2016 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI 2016)*, Baden-Baden, Germany, Sep. 2016.

[25] C. N. Vasconcelos and B. Rosenhahn, "Bipartite Graph Matching Computation on GPU," in *International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition*. Springer, 2009, pp. 42–55.

[26] D. W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," *Journal of the society for Industrial and Applied Mathematics*, vol. 11, no. 2, pp. 431–441, 1963.

[27] J. J. Moré and D. C. Sorensen, "Computing a Trust Region Step," *SIAM Journal on Scientific and Statistical Computing*, vol. 4, no. 3, pp. 553–572, 1983.