Stability Analysis of Polytopic Markov Jump Linear Systems with Applications to Sequence-Based Control over Networks

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Abstract: This paper deals with sequence-based control over networks with time-varying and generally unknown delay and loss probabilities. We show that the problems of stability analysis and controller synthesis can be addressed using a polytopic Markov jump linear system describing an augmented system. For this kind of systems, we derive a necessary and sufficient condition for mean square stability that extends existing results in literature. Likewise, we provide a sufficient condition for mean square stabilizability in terms of an LMI feasibility test. The results are illustrated in a numerical example.

Keywords: Control over networks, Control under communication constraints, Control of switched systems, Stability and stabilization of hybrid systems

1. INTRODUCTION

Plenty of research has been carried out to date focusing on control in networked environments such as today's cyber-physical systems (CPS). Well known examples are smart grids, water distribution systems, or intelligent manufacturing systems (Hu et al., 2016; Zhong et al., 2017).

To exchange data such as system states, control commands, or sensor readings, such kind of systems typically make use of off-the-shelf communication infrastructure (Hehenberger et al., 2016; Karnouskos et al., 2019). Main benefits are increased flexibility and simplified installation and maintenance compared to traditional point-to-point connections between the individual components. On the other hand, such networks are prone to non-deterministic communication delays and data losses, which, in turn, can severely degrade the possible control performance (Lucia et al., 2016).

To cope with packet delays and losses, network-aware controllers commonly transmit not only the control command for the current time step to the plant but also a sequence of predictive inputs for the next, say N, time steps. This technique, known as *sequence-based control* in literature, has been considered in a multitude of works in the last years.

Research effort in this regard has led to a variety of different approaches for the computation of such control sequences, ranging from methods building on nominal controllers that completely disregard the underlying network, e.g., Bemporad (1998); Liu (2010), to more sophisticated approaches based on model predictive and optimal control in, for instance, Reble et al. (2013); Quevedo and Jurado (2014); Fischer et al. (2013); Jurado et al. (2015); Dolgov et al. (2015); Li et al. (2016); Rosenthal et al. (2019).

To reflect the randomness of packet delays and losses in real networks, it is common to treat them as realizations of a stochastic process. In this regard, a typical assumption is that this process is stationary with known statistics. Yet, when communication resources are shared without statically configured scheduling, the provided quality of service fluctuates, leading to changing communication conditions that are hardly completely foreseen in advance. Control algorithms that are able to cope with varying packet delays or data rates while at the same time guaranteeing a certain control performance are thus a fundamental building block of flexible and future-proof CPS. Consequently, research in this regard demands further attention.

In this paper, we are concerned with sequence-based control in settings where packet delays and losses are governed by a non-stationary stochastic process with unknown statistics. Focusing on linear time-invariant plants that are subject to white second order wide-sense stationary noise, the contributions of this work are as follows.

We first develop a combined model that jointly expresses the plant dynamics and the underlying network in terms of a discrete-time Markov jump linear system (MJLS). This is similar to what was done in, e.g., Fischer et al. (2013); Rosenthal et al. (2019); Rosenthal and Hanebeck (2019), where fixed packet delay and loss probabilities were considered. However, in contrast to these works, the resulting MJLS turns out to be *polytopic*, that is, the Markov chain governing the switching of the modes is time-inhomogeneous and its transition matrix varies within a convex polytope.

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Second, we prove that the necessary and sufficient condition for mean square stability of noise-free polytopic MJLS and polytopic MJLS with bounded noise presented in Lun et al. (2016) and Lun et al. (2017, 2019), respectively, which requires the calculation of the *joint spectral radius (JSR)* of a set of matrices, also holds true for polytopic MJLS driven by wide-sense stationary noise.

Finally, we derive a sufficient condition for the existence of a stabilizing mode-independent state feedback controller in terms of a *linear matrix inequality (LMI)* feasibility problem similar to the ones presented in Aberkane (2011) and Zhang and Boukas (2009) for polytopic MJLS without noise and MJLS with partly unknown transition probabilities, respectively.

Notation Throughout this paper, vectors will be indicated by underlined letters ($\underline{x}$) and boldface capital letters indicate matrices, e.g., $\mathbf{A}$. We use $\mathbf{I}_n$ to denote the n-dimensional identity matrix, $\mathbf{0}$ to denote zero matrices of arbitrary dimension, and a subscript k, e.g., $\underline{x}_k$, to indicate the time step. Transposition of a vector or a matrix is indicated by $\underline{x}^{\mathrm{T}}$ and $\mathbf{A}^{\mathrm{T}}$, and $\mathbf{A} > 0$ ($\mathbf{A} \geq 0$) means that the matrix $\mathbf{A}$ is positive definite (positive semidefinite). The Kronecker product of two matrices $\mathbf{A}$ and $\mathbf{B}$ is denoted by $\mathbf{A} \otimes \mathbf{B}$, $\mathrm{vec}\,(\mathbf{A})$ denotes vectorization, and $\mathrm{diag}(\mathbf{A}_1, \dots, \mathbf{A}_i)$ denotes the block-diagonal matrix formed by the matrices $\mathbf{A}_1, \dots, \mathbf{A}_i$. Furthermore, we denote by $\mathrm{conv}(\mathcal{M})$ the convex hull of a set $\mathcal{M}$. Finally, $\mathbf{A}_{:j}$ indicates the j-th column of $\mathbf{A}$ and $\mathbb{1}_{i=j}$ is the indicator function, i.e., $\mathbb{1}_{i=j} = 1$ if $i = j$ and 0 otherwise.

2. PROBLEM FORMULATION

Consider a linear time-invariant plant with discrete-time dynamics given by

$$\underline{x}_{k+1} = \mathbf{A}\underline{x}_k + \mathbf{B}\underline{u}_k + \underline{w}_k\,, \tag{1}$$

with $\underline{x}_k \in \mathbb{R}^n$ the plant state, $\underline{u}_k \in \mathbb{R}^l$ the control input, $\mathbf{A} \in \mathbb{R}^{n \times n}$ the system matrix and $\mathbf{B} \in \mathbb{R}^{n \times l}$ the input matrix. The process noise $\underline{w}_k$ is a white, zero mean wide-sense stationary sequence with covariance matrix $\mathbf{W}$. The initial plant state $\underline{x}_0$ is assumed to be Gaussian with given mean $\hat{\underline{x}}_0$ and covariance $\mathbf{X}_0$, and independent of $\underline{w}_k$ for all k.

The control inputs are computed by a remote controller and then sent to the actuator, which is attached to the plant, using a packet-based network. Due to the nature of the network, packets containing control inputs can randomly experience (potentially unbounded) delays or get lost. We interpret packet losses as infinite delays, so that we can model the delay of a packet transmitted to the actuator at time step k by the random variable $\tau_k \in \mathbb{N}$ with corresponding probability mass function (pmf) f_k. Consequently, $f_k(t)$ denotes the probability that the packet sent from the controller to the actuator at time k arrives at time step $k + t$ (i.e., $f_k(t) = \Pr[\tau_k = t]$), whereas the probability that this packet gets lost is given by $f_k(\infty)$.

Finally, we assume that any two τ_k and $\tau_{\tilde{k}}$ are mutually independent for $k \neq \tilde{k}$, i.e., packet delays and losses are independent over time. However, to reflect the nature of modern communication networks, we assume that the probabilities for packet delays and losses may change over time, and are not known in advance. That is, the pmfs f_k are not known at design time.

To cope with packet delays and losses, the controller is sequence-based and transmits predicted control inputs for the next N time steps in conjunction with the one for the current time step. Hence, at each time step, the data packet transmitted to the actuator consists of a sequence of $N+1$ control inputs

$$\underline{U}_k = \left[\underline{u}_{k|k}^{\mathrm{T}}\ \underline{u}_{k+1|k}^{\mathrm{T}}\ \cdots\ \underline{u}_{k+N|k}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{(N+1)l}\,,$$

with $\underline{u}_{k+i|k}$ the control input computed at time k to be applied at time $k+i$, $i = 0, \dots, N$. At the plant side, only the most recent control sequence is maintained, whereas all others are discarded upon reception. The inputs provided by this sequence are then applied to the plant one after another until a newer sequence is received.

Due to this buffering procedure, at any time k the actually applied control input must stem from one of the sequences $\underline{U}_{k-N}, \underline{U}_{k-N+1}, \dots, \underline{U}_k$. If none of these sequences is buffered at the actuator side, but for instance an older one, no applicable input is present, and the default input $\underline{u}_k^{df} = \underline{0}$ is applied to the plant, that is, the plant runs open loop. This can only happen if all of the packets containing the aforementioned sequences get lost during the transmission or suffer large delays. The probability that the default input is applied is thus not only determined by the packet loss rate, but depends also on the sequence length chosen by the designer. In this regard, the following assumption is justified to ensure that the controller can influence the plant behavior.

Assumption 1. The length of the control sequences is chosen such that the probability of two consecutive applications of the default input is less than one.

We aim to find stability conditions for networked control systems that operate in the given setup and, moreover, seek to synthesize a linear state feedback control law that is able to stabilize the plant. More precisely, we will use the notion of mean square stability as defined below in Definition 4.

3. PRELIMINARIES AND BASIC RESULTS

As mentioned above, at any time k the actually applied control input is either the default input ($\underline{u}_k^{df} = \underline{0}$) or is part of one of the sequences $\underline{U}_{k-N}, \underline{U}_{k-N+1}, \dots, \underline{U}_k$.

This observation is the key to obtain the MJLS that jointly expresses the plant dynamics (1) and the buffering procedure employed by the actuator and can be formalized as follows. First, we introduce a vector $\underline{\eta}_k$ that contains all inputs from *past* control sequences that are still applicable at time k or later according to

$$\underline{\eta}_k = \begin{bmatrix} \left[\underline{u}_{k|k-1}^{\mathrm{T}}\ \underline{u}_{k+1|k-1}^{\mathrm{T}}\ \cdots\ \underline{u}_{k+N-1|k-1}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \left[\underline{u}_{k|k-2}^{\mathrm{T}}\ \underline{u}_{k+1|k-2}^{\mathrm{T}}\ \cdots\ \underline{u}_{k+N-2|k-2}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \vdots \\ \left[\underline{u}_{k|k-N+1}^{\mathrm{T}}\ \underline{u}_{k+1|k-N+1}^{\mathrm{T}}\right]^{\mathrm{T}} \\ \underline{u}_{k|k-N} \end{bmatrix} \in \mathbb{R}^d\,,$$

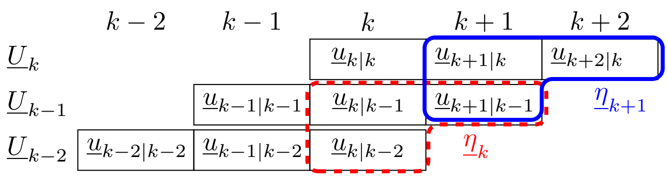

Fig. 1. Visualization of $\underline{\eta}_{k+1}$, $\underline{\eta}_k$, $\underline{U}_k$, and their relationship for $N = 2$.

where $d = \frac{lN(N+1)}{2}$. The corresponding dynamics is

$$\underline{\eta}_{k+1} = \mathbf{F}\underline{\eta}_k + \mathbf{G}\underline{U}_k\,, \tag{2}$$

with $\mathbf{F} \in \mathbb{R}^{d\times d}$ and $\mathbf{G} \in \mathbb{R}^{d\times(N+1)l}$ given by

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{(N-1)l} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{(N-2)l} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{I}_l & \mathbf{0} \end{bmatrix}, \; \mathbf{G} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{Nl} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

The relationship between $\underline{\eta}_{k+1}$, $\underline{\eta}_k$, and $\underline{U}_k$ according to (2) is visualized for $N = 2$ in Figure 1.

Together with a scalar variable θ_k, defined according to

$$\theta_k = \begin{cases} i & \text{if } \underline{u}_k \text{ is part of } \underline{U}_{k-i} \\ N+1 & \text{if } \underline{u}_k = \underline{u}_k^{df} \end{cases}, \tag{3}$$

with $i \in \{0, 1, \dots, N\}$ and

$$\mathbf{H}^{(\theta_k)} = [\mathbb{1}_{\theta_k=1}\mathbf{I}_l \;\mathbf{0}\; \mathbb{1}_{\theta_k=2}\mathbf{I}_l \;\mathbf{0}\; \dots\; \mathbb{1}_{\theta_k=N}\mathbf{I}_l]\,,$$
$$\mathbf{J}^{(\theta_k)} = [\mathbb{1}_{\theta_k=0}\mathbf{I}_l \;\mathbf{0}]\,,$$

the actually applied input can then be written as

$$\underline{u}_k = \mathbf{H}^{(\theta_k)}\underline{\eta}_k + \mathbf{J}^{(\theta_k)}\underline{U}_k\,.$$

It is clear from (3) that the set of possible values of θ_k is directly related to the sequence length. For $i \in \{0, 1, \dots, N\}$, $\theta_k = i$ corresponds to the *age* of the buffered sequence, whereas $\theta_k = N + 1$ indicates an empty buffer at the actuator side. Moreover, we have the following result.

Theorem 2. The process $\{\theta_k\}$ forms a time-inhomogeneous Markov chain with transition probabilities $p_{k,ij} = \Pr[\theta_{k+1} = j|\theta_k = i]$ determined by the pmfs f_k according to

$$p_{k,ij} = \begin{cases} q_k & \text{if } j = 0 \\ (1-q_k)\prod_{m=0}^{i-1}(1-\tilde{q}_k(m)) & \text{if } j = i+1 \\ 0 & \text{if } j > i+1 \\ (1-q_k)\tilde{q}_k(j-1)\prod_{m=0}^{j-2}(1-\tilde{q}_k(m)) & \text{if } 1 \le j \le i \le N \\ (1-q_k)\prod_{m=0}^{N-1}(1-\tilde{q}_k(m)) & \text{if } i = j = N+1 \end{cases},$$

with $q_k = f_{k+1}(0)$ and

$$\tilde{q}_k(j) = \frac{f_{k-j}(j+1)}{1-\sum_{m=0}^{j} f_{k-j}(m)}\,, \tag{4}$$

denoting the probability that $\underline{U}_{k-j}$ arrives at time $k+1$ given that it has not been received at time k or earlier.

Proof. The proof is similar to the time-homogeneous case discussed in Fischer et al. (2013) and exploits that packet delays are assumed to be independent. First, $j = 0$ indicates a transition from $\theta_k = i$ to $\theta_{k+1} = 0$, which means that $\underline{U}_{k+1}$ arrives at the actuator without delay. The corresponding probability is thus $f_{k+1}(0)$.

For the remaining cases, we first conclude from (4) that $1-\tilde{q}_k(j)$ denotes the probability that $\underline{U}_{k-j}$ does not arrive at the actuator at time $k+1$, given that is has not arrived earlier. Then we note that a transition from $\theta_k = i$ to $\theta_{k+1} = i+1$ corresponds to the event that the currently buffered sequence $\underline{U}_{k-i}$ is not replaced by a newer one which means that none of the sequences $\underline{U}_{k-i+1}, \dots, \underline{U}_{k+1}$ will arrive at $k+1$. Consequently,

$$p_{k,i(i+1)} = (1-q_k)\prod_{m=0}^{i-1}(1-\tilde{q}_k(m))\,.$$

Transitions from $\theta_k = i$ to $\theta_{k+1} \ge i+2$ are impossible since the age of the buffered sequence can only increase by one, namely in case it is not replaced. Hence, $p_{k,ij} = 0$ for $j > i+1$.

For $1 \le j \le i \le N$, we have transitions from $\theta_k = i$ to $\theta_{k+1} = j$ that indicate a replacement of $\underline{U}_{k-i}$ by a newer sequence $\underline{U}_{k-(j-1)}$. The corresponding probability is given by

$$p_{k,ij} = (1-q_k)\tilde{q}_k(j-1)\prod_{m=0}^{j-2}(1-\tilde{q}_k(m))\,.$$

Finally, we note that the case $i = j = N+1$ corresponds to the event that at time $k+1$ no valid packet is buffered given that no valid packet was buffered at time k either. Hence,

$$p_{k,(N+1)(N+1)} = (1-q_k)\prod_{m=0}^{N-1}(1-\tilde{q}_k(m))\,,$$

which concludes the proof.

Remark 3. The result implies that the corresponding transition matrices $\mathbf{P}_k$ are lower Hessenberg matrices and that the last two rows are always equal. For example, for $N = 2$ we have

$$\mathbf{P}_k = \begin{bmatrix} q_k & 1-q_k & 0 & 0 \\ q_k & (1-q_k)\tilde{q}_k(0) & (1-q_k)(1-\tilde{q}_k(0)) & 0 \\ q_k & (1-q_k)\tilde{q}_k(0) & (1-q_k)\tilde{q}_k(1)(1-\tilde{q}_k(0)) & (1-q_k)\prod_{m=0}^{1}(1-\tilde{q}_k(m)) \\ q_k & (1-q_k)\tilde{q}_k(0) & (1-q_k)\tilde{q}_k(1)(1-\tilde{q}_k(0)) & (1-q_k)\prod_{m=0}^{1}(1-\tilde{q}_k(m)) \end{bmatrix}.$$

By defining $\underline{\psi}_k = \left[\underline{x}_k^{\mathrm{T}} \; \underline{\eta}_k^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^s$, $s = n + d$, we obtain the augmented dynamics

$$\underline{\psi}_{k+1} = \tilde{\mathbf{A}}^{(\theta_k)}\underline{\psi}_k + \tilde{\mathbf{B}}^{(\theta_k)}\underline{U}_k + \underline{\tilde{w}}_k\,, \tag{5}$$

where

$$\tilde{\mathbf{A}}^{(\theta_k)} = \begin{bmatrix} \mathbf{A} & \mathbf{B}\mathbf{H}^{(\theta_k)} \\ \mathbf{0} & \mathbf{F} \end{bmatrix}, \tilde{\mathbf{B}}^{(\theta_k)} = \begin{bmatrix} \mathbf{B}\mathbf{J}^{(\theta_k)} \\ \mathbf{G} \end{bmatrix}, \underline{\tilde{w}}_k = \begin{bmatrix} \underline{w}_k \\ \underline{0} \end{bmatrix},$$

$\mathrm{E}\{\underline{\tilde{w}}_k\} = \underline{0}$, and $\mathrm{Cov}\{\underline{\tilde{w}}_k\} = \tilde{\mathbf{W}} = \mathrm{diag}(\mathbf{W}, \mathbf{0})$.

Eq. (5) represents the desired MJLS, with the jumping between the $N+2$ modes governed by the time-inhomogeneous Markov chain $\{\theta_k\}$. We note that the true value of θ_k is only known to the controller if acknowledgments sent back from the actuator are delivered instantaneously and without losses. This assumption, often referred to as *TCP-like* communication in literature, does not hold in real networks since each data packet is subject to physical constraints such as processing and propagation speed. Thus, in realistic environments, θ_k is either unknown or becomes available only belatedly.

Since the pmfs f_k are unknown, the transition probabilities cannot be calculated so that, apart from the zero entries in the upper right corner, all entries of $\mathbf{P}_k$ are generally unknown. However, Assumption 1 implies that $p_{k,(N+1)(N+1)} \in [0, \delta]$ for some $\delta < 1$. Exploiting the dependencies between the entries according to Theorem 2, we can use the multi-simplex approach from Oliveira et al. (2008) to show that for any N we can always find an $L \in \mathbb{N}$ and corresponding transition matrices $\mathbf{T}(1), \ldots, \mathbf{T}(L)$ such that at each time step $\mathbf{P}_k$ can be expressed in terms of a convex combination

$$\mathbf{P}_k = \sum_{r=1}^{L} \alpha_k(r)\mathbf{T}(r) , \quad \alpha_k(r) \geq 0 , \quad \sum_{r=1}^{L} \alpha_k(r) = 1 .$$

In other words, $\mathbf{P}_k$ varies within a convex polytope with vertices $\mathbf{T}(1), \ldots, \mathbf{T}(L)$ and each transition probability is given by $p_{k,ij} = \sum_{r=1}^{L} \alpha_k(r) t_{ij}(r)$. For instance, for $N = 2$, $\mathbf{P}_k$ varies within a polytope with $L = 36$ vertices, while for $N = 5$ we already have $L = 8640$.

For the subsequent analyses of the polytopic MJLS (5), we use the following definition of mean square stability that can, for instance, be found in Costa and Fragoso (1993).

Definition 4. The dynamics (5) with $\underline{U}_k \equiv \underline{0}$ is *mean square stable* if for every initial condition $(\underline{\psi}_0, \theta_0)$ it holds

$$\lim_{k\to\infty} \left\| \mathrm{E}\left\{ \underline{\psi}_k \right\} \right\| = 0 , \quad \lim_{k\to\infty} \left\| \mathrm{E}\left\{ \underline{\psi}_k \underline{\psi}_k^{\mathrm{T}} \right\} - \mathbf{Q} \right\| = 0 , \quad (6)$$

for some $\mathbf{Q}$ that is independent of the initial conditions.

Following Costa et al. (2006) and Lun et al. (2016), we introduce some notation associated with the augmented dynamics that will be useful in the remainder according to

$$\tilde{\mathbf{Q}}_k^{(i)} = \mathrm{E}\left\{ \underline{\psi}_k \underline{\psi}_k^{\mathrm{T}} \mathbb{1}_{\theta_k = i} \right\} , \quad (7)$$

$$\tilde{\mathbf{W}}_k^{(i)} = \mathrm{E}\left\{ \underline{\tilde{w}}_k \underline{\tilde{w}}_k^{\mathrm{T}} \mathbb{1}_{\theta_k = i} \right\} , \quad (8)$$

$$\underline{\phi}(\tilde{\mathbf{Q}}_k) = \left[\mathrm{vec}\left(\tilde{\mathbf{Q}}_k^{(0)} \right)^{\mathrm{T}} \ \ldots \ \mathrm{vec}\left(\tilde{\mathbf{Q}}_k^{(N+1)} \right)^{\mathrm{T}} \right]^{\mathrm{T}} , \quad (9)$$

$$\underline{\phi}(\tilde{\mathbf{W}}_k) = \left[\mathrm{vec}\left(\tilde{\mathbf{W}}_k^{(0)} \right)^{\mathrm{T}} \ \ldots \ \mathrm{vec}\left(\tilde{\mathbf{W}}_k^{(N+1)} \right)^{\mathrm{T}} \right]^{\mathrm{T}} , \quad (10)$$

$$\underline{\mathbf{A}} = \mathrm{diag}(\tilde{\mathbf{A}}^{(0)} \otimes \tilde{\mathbf{A}}^{(0)}, \ldots, \tilde{\mathbf{A}}^{(N+1)} \otimes \tilde{\mathbf{A}}^{(N+1)}) .$$

Using (5) and (8), it follows that the dynamics of the mode-conditioned second moment (7) is

$$\tilde{\mathbf{Q}}_{k+1}^{(j)} = \sum_{i=0}^{N+1} \sum_{r=1}^{L} \alpha_k(r) t_{ij}(r) \left(\tilde{\mathbf{A}}^{(i)} \tilde{\mathbf{Q}}_k^{(i)} \left(\tilde{\mathbf{A}}^{(i)} \right)^{\mathrm{T}} + \tilde{\mathbf{W}}_k^{(i)} \right) ,$$

and consequently

$$\mathrm{vec}\left(\tilde{\mathbf{Q}}_{k+1}^{(j)} \right) = \sum_{i=0}^{N+1} \sum_{r=1}^{L} \alpha_k(r) t_{ij}(r) \left(\tilde{\mathbf{A}}^{(i)} \otimes \tilde{\mathbf{A}}^{(i)} \right) \mathrm{vec}\left(\tilde{\mathbf{Q}}_k^{(i)} \right) + \sum_{i=0}^{N+1} \sum_{r=1}^{L} \alpha_k(r) t_{ij}(r) \, \mathrm{vec}\left(\tilde{\mathbf{W}}_k^{(i)} \right) ,$$

since vectorization is a linear operation and it holds $\mathrm{vec}\left(\mathbf{XYZ}\right) = \left(\mathbf{Z}^{\mathrm{T}} \otimes \mathbf{X}\right) \mathrm{vec}\left(\mathbf{Y}\right)$. Writing this in terms of $\underline{\phi}(\tilde{\mathbf{Q}}_k)$ and $\underline{\phi}(\tilde{\mathbf{W}}_k)$ yields

$$\mathrm{vec}\left(\tilde{\mathbf{Q}}_{k+1}^{(j)} \right) = \sum_{r=1}^{L} \alpha_k(r) \left(\mathbf{T}_{:j}(r)^{\mathrm{T}} \otimes \mathbf{I}_{s^2} \right) \left(\underline{\mathbf{A}} \underline{\phi}(\tilde{\mathbf{Q}}_k) + \underline{\phi}(\tilde{\mathbf{W}}_k) \right) ,$$

so that the dynamics of (9) is finally given by

$$\underline{\phi}(\tilde{\mathbf{Q}}_{k+1}) = \mathbf{\Gamma}_k \underline{\phi}(\tilde{\mathbf{Q}}_k) + \mathbf{\Sigma}_k \underline{\phi}(\tilde{\mathbf{W}}_k) , \quad (11)$$

where

$$\mathbf{\Gamma}_k = \sum_{r=1}^{L} \alpha_k(r) \left(\mathbf{T}(r)^{\mathrm{T}} \otimes \mathbf{I}_{s^2} \right) \underline{\mathbf{A}} , \quad (12)$$

$$\mathbf{\Sigma}_k = \sum_{r=1}^{L} \alpha_k(r) \mathbf{T}(r)^{\mathrm{T}} \otimes \mathbf{I}_{s^2} . \quad (13)$$

Introducing the finite set $\mathcal{A}_L = \{\mathbf{\Lambda}(1), \mathbf{\Lambda}(2) \ldots, \mathbf{\Lambda}(L)\}$ with $\mathbf{\Lambda}(r) = \left(\mathbf{T}(r)^{\mathrm{T}} \otimes \mathbf{I}_{s^2}\right) \underline{\mathbf{A}}$, we get from (12) that $\mathbf{\Gamma}_k \in \mathrm{conv}(\mathcal{A}_L)$.

With these prerequisites, we can now establish a necessary and sufficient condition for mean square stability of (5) based on the joint spectral radius (JSR) of $\mathcal{A}_L$.

Generally, for a given set of matrices, the JSR is defined as follows according to Jungers (2009).

Definition 5. Let $\mathcal{M} = \{\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_M\}$ be a set of real square matrices and set

$$\Pi_k(\mathcal{M}) = \left\{ \prod_{i=1}^{k} \mathbf{M}_{l_i} | l_1, l_2, \ldots, l_k \in \{1, \ldots, M\} \right\} .$$

For any matrix norm, consider the quantity

$$\rho_k(\mathcal{M}) = \sup_{\mathbf{P} \in \Pi_k(\mathcal{M})} \|\mathbf{P}\|^{1/k} . \quad (14)$$

The limit for $k \to \infty$ of this quantity, which always exists and is independent of the chosen norm,

$$\rho(\mathcal{M}) = \lim_{k\to\infty} \rho_k(\mathcal{M}) ,$$

is called the *joint spectral radius* of $\mathcal{M}$.

In the remainder, we will make use of the following two well-known facts. Proofs are, for instance, given in Berger and Wang (1992) and Jungers (2009), respectively.

Fact 6. For any finite set of matrices $\mathcal{M}$, $\rho(\mathcal{M}) < 1$ if and only if any $\mathbf{P} \in \Pi_k(\mathcal{M})$ converges to $\mathbf{0}$ as $k \to \infty$.

Fact 7. For any finite set of matrices $\mathcal{M}$, it holds $\rho(\mathcal{M}) = \rho(\mathrm{conv}(\mathcal{M}))$.

Before we proceed with the presentation of the main results of this work in the next section, we want to emphasize that the assumption of independent packet delays and losses is essential for the augmented dynamics (5) to be a MJLS. If instead correlations between consecutive delays and losses are considered, $\{\theta_k\}$ is generally *not* a Markov chain. This is due to the fact that θ_k depends on the delays experienced by the control sequences $\underline{U}_{k-N}, \ldots, \underline{U}_k$ (cf. (3)) and thus on the random variables $\tau_{k-N}, \ldots, \tau_k$. For instance, $\theta_k = 2$ if and only if $\tau_{k-2} \leq 2, \tau_{k-1} > 1$, and $\tau_k > 0$. For the common assumption of Markovian packet delays and losses this dependency for example leads to $\Pr[\theta_{k+1} = 1 | \theta_k = 1, \theta_{k-1} = 0] \neq \Pr[\theta_{k+1} = 1 | \theta_k = 1, \theta_{k-1} = 1]$, certifying that $\{\theta_k\}$ is indeed not a Markov chain.

However, the modeling approach used in this paper resembles the one used by Quevedo and Gupta (2011) and Quevedo et al. (2015) in their research on sequence-based anytime control with Markovian processor availability. By adapting the methodology used therein, we expect that we can construct an aggregated Markov chain of the form $\{(\tau_k, \theta_k)\}$ so that the augmented dynamics would then again be expressed in terms of a polytopic MJLS. Just very recently, a similar idea has been seized by Lun and D'Innocenzo (2019) in the context of wireless networked control. This would in turn mean that the results presented in the following would carry over with only little modification to the case of Markovian packet delays and losses with time-varying and unknown transition probabilities. Substantiating this proposition is part of our ongoing research.

4. MAIN RESULTS

Theorem 8. The polytopic MJLS with wide-sense stationary noise as given by (5) (with $\underline{U}_k \equiv \underline{0}$) is mean square stable if and only if $\rho(\mathcal{A}_L) < 1$.

Proof. From (11) it follows that

$$\underline{\phi}(\tilde{\mathbf{Q}}_k) = \mathbf{\Gamma}_{k-1}\mathbf{\Gamma}_{k-2}\dots\mathbf{\Gamma}_0\underline{\phi}(\tilde{\mathbf{Q}}_0) + \sum_{i=0}^{k-1}\left(\prod_{l=i+1}^{k-1}\mathbf{\Gamma}_l^{\mathrm{T}}\right)^{\mathrm{T}}\mathbf{\Sigma}_i\underline{\phi}(\tilde{\mathbf{W}}_i)\,. \quad (15)$$

To show necessity, the hypothesis (6) implies that there exist $\mathbf{Q}$ such that for any initial condition

$$\lim_{k\to\infty}\mathrm{E}\left\{\underline{\psi}_k\underline{\psi}_k^{\mathrm{T}}\right\} = \lim_{k\to\infty}\sum_{i=0}^{N+1}\tilde{\mathbf{Q}}_k^{(i)} = \mathbf{Q}\,,$$

which means that each $\tilde{\mathbf{Q}}_k^{(i)}$ must converge to some $\mathbf{Q}^{(i)}$. Consequently, we have for any initial condition $\underline{\phi}(\tilde{\mathbf{Q}}_0)$ that

$$\lim_{k\to\infty}\underline{\phi}(\tilde{\mathbf{Q}}_k) = \left[\mathrm{vec}\left(\mathbf{Q}^{(0)}\right)^{\mathrm{T}}\ \dots\ \mathrm{vec}\left(\mathbf{Q}^{(N+1)}\right)^{\mathrm{T}}\right]^{\mathrm{T}}. \quad (16)$$

For $\underline{\phi}(\tilde{\mathbf{Q}}_0) = \underline{0}$, i.e., $\mathrm{E}\left\{\underline{\psi}_0\underline{\psi}_0^{\mathrm{T}}\right\} = \mathbf{0}$, the first term on the right side of (15) vanishes for any k, implying that the second term converges to the limit given in (16). Since this term is independent of $\underline{\phi}(\tilde{\mathbf{Q}}_0)$, the first term on the right side of (15) must vanish for any initial condition. Thus, $\mathbf{\Gamma}_{k-1}\mathbf{\Gamma}_{k-2}\dots\mathbf{\Gamma}_0 \in \Pi_k(\mathrm{conv}(\mathcal{A}_L))$ must converge to $\mathbf{0}$. Applying Facts 6 and 7 then yields that $\rho(\mathcal{A}_L) < 1$.

For sufficiency, we first note, using again Facts 6 and 7, that $\rho(\mathcal{A}_L) < 1$ implies that the first term on the right side of (15) vanishes as $k \to \infty$. Then, according to (13) the Frobenius norm of $\mathbf{\Sigma}_k$ is bounded from above by some $\xi > 0$

$$\|\mathbf{\Sigma}_k\|_F \le \sum_{r=1}^{L}\alpha_k(r)\left\|\mathbf{T}(r)^{\mathrm{T}}\otimes\mathbf{I}_{s^2}\right\|_F \le \sum_{r=1}^{L}\left\|\mathbf{T}(r)^{\mathrm{T}}\otimes\mathbf{I}_{s^2}\right\|_F = \xi\,.$$

Similarly, from (10) we get for $\underline{\phi}(\tilde{\mathbf{W}}_k)$

$$\|\underline{\phi}(\tilde{\mathbf{W}}_k)\|_2 = \sqrt{\sum_{i=0}^{N+1}\left\|\mathrm{vec}\left(\tilde{\mathbf{W}}_k^{(i)}\right)\right\|_2^2} = \sqrt{\sum_{i=0}^{N+1}\left\|\tilde{\mathbf{W}}_k^{(i)}\right\|_F^2} = \sqrt{\sum_{i=0}^{N+1}\|\mathrm{E}\{\mathbb{1}_{\theta_k=i}\}\|^2\,\|\tilde{\mathbf{W}}\|_F^2} \le \|\tilde{\mathbf{W}}\|_F\,.$$

Thus, for the norm of the sum on the right of (15) we obtain

$$\left\|\sum_{i=0}^{k-1}\left(\prod_{l=i+1}^{k-1}\mathbf{\Gamma}_l^{\mathrm{T}}\right)^{\mathrm{T}}\mathbf{\Sigma}_i\underline{\phi}(\tilde{\mathbf{W}}_i)\right\|_2 \le \xi\|\tilde{\mathbf{W}}\|_F\sum_{i=0}^{k-1}\left\|\left(\prod_{l=i+1}^{k-1}\mathbf{\Gamma}_l^{\mathrm{T}}\right)^{\mathrm{T}}\right\|_F.$$

Since $\rho(\mathcal{A}_L) < 1$, it follows from (14) that for sufficiently large k we have $\rho_k(\mathrm{conv}(\mathcal{A}_L)) < \beta < 1$. Thus, using the same arguments as in Lun et al. (2016), we can state that there exist $\chi \ge 1$ such that for all k it holds $\|\mathbf{P}\|_F \le \chi\beta^k$ for any $\mathbf{P} \in \Pi_k(\mathrm{conv}(\mathcal{A}_L))$. Hence,

$$\xi\|\tilde{\mathbf{W}}\|_F\sum_{i=0}^{k-1}\left\|\left(\prod_{l=i+1}^{k-1}\mathbf{\Gamma}_l^{\mathrm{T}}\right)^{\mathrm{T}}\right\|_F \le \xi\chi\|\tilde{\mathbf{W}}\|_F\sum_{i=0}^{k-1}\beta^{k-i-1} < \infty\,,$$

as the sum on the right is a geometric series, which establishes the convergence of the second moment. To conclude the proof, it remains to show that $\rho(\mathcal{A}_L) < 1 \Rightarrow \lim_{k\to\infty}\left\|\mathrm{E}\left\{\underline{\psi}_k\right\}\right\|_2 = 0$. Similar to Costa and Fragoso (1993), this is readily verified by considering the noise free-dynamics $\underline{\psi}_{k+1} = \tilde{\mathbf{A}}^{(\theta_k)}\underline{\psi}_k$ for which $\left\|\tilde{\mathbf{Q}}_k^{(i)}\right\|_F \to 0$ as k approaches infinity and noticing that

$$\left\|\mathrm{E}\left\{\underline{\psi}_k\right\}\right\|_2^2 \le \mathrm{E}\left\{\left\|\underline{\psi}_k\right\|_2^2\right\} \le s\sum_{i=0}^{N+1}\left\|\tilde{\mathbf{Q}}_k^{(i)}\right\|_F.$$

It has been proved in Lun et al. (2016) that determining whether $\rho(\mathcal{A}_L) < 1$ is NP-hard even for polytopic MJLS with only two vertices in $\mathcal{A}_L$. Thus, unless $P = NP$, there cannot exist any polynomial time algorithm to decide whether (5) is mean square stable or not.

In the remainder we provide sufficient conditions for mean square stability and stabilizability, respectively, based on LMI feasibility problems that are similar to, but more general than the ones derived in Aberkane (2011) for polytopic MJLS without noise. To that end, we first establish a sufficient condition for mean square stability that requires that an infinite set of matrix inequalities be solved.

Theorem 9. If there exist $\mathbf{D}^{(0)}, \mathbf{D}^{(1)}, \dots, \mathbf{D}^{(N+1)} > 0$ such that for $i = 0, 1, \dots, N+1$ it holds

$$\mathbf{D}^{(i)} - \left(\tilde{\mathbf{A}}^{(i)}\right)^{\mathrm{T}}\sum_{j=0}^{N+1}\sum_{r=1}^{L}\alpha_k(r)t_{ij}(r)\mathbf{D}^{(j)}\tilde{\mathbf{A}}^{(i)} > 0\,, \quad (17)$$

then the polytopic MJLS (5) (with $\underline{U}_k \equiv \underline{0}$) is mean square stable.

Proof. We proceed similar to the proof of Proposition 7 in Costa and Fragoso (1993) and consider the dynamics

$$\mathbf{Y}_{k+1}^{(j)} = \sum_{i=0}^{N+1}\sum_{r=1}^{L}\alpha_k(r)t_{ij}(r)\tilde{\mathbf{A}}^{(i)}\mathbf{Y}_k^{(i)}\left(\tilde{\mathbf{A}}^{(i)}\right)^{\mathrm{T}}, \quad (18)$$

with initial condition $\mathbf{Y}_0^{(i)} \ge 0$ for $i = 0, \dots, N+1$. By noticing that

$$\mathcal{V}(\mathbf{Y}_k) = \sum_{i=0}^{N+1}\mathrm{tr}[\mathbf{D}^{(i)}\mathbf{Y}_k^{(i)}]\,,$$

is a Lyapunov function for the system (18) and, moreover, is radially unbounded, we conclude that the equilibrium

$\mathbf{Y}^{(i)} = \mathbf{0}$ is globally asymptotically stable. Consequently, the vectorized dynamics

$$\underline{\phi}\left(\mathbf{Y}_{k+1}\right) = \mathbf{\Gamma}_k \underline{\phi}\left(\mathbf{Y}_k\right) ,$$

with $\mathbf{\Gamma}_k$ as given by (12), is also globally asymptotically stable. Thus, $\mathbf{\Gamma}_k \mathbf{\Gamma}_{k-1} \ldots \mathbf{\Gamma}_0 \in \Pi_k(\mathrm{conv}(\mathcal{A}_L))$ must converge to $\mathbf{0}$ for any initial condition, from which the claim follows.

Based on (17), the following result provides a sufficient condition for mean square stability of (5) that is easy to evaluate using state-of-the-art LMI solvers.

Theorem 10. There exist $\mathbf{D}^{(0)}, \mathbf{D}^{(1)}, \ldots, \mathbf{D}^{(N+1)} > 0$ satisfying (17) for $i = 0, 1, \ldots, N+1$ if and only if there exist $\mathbf{S}^{(0)}, \mathbf{S}^{(1)}, \ldots, \mathbf{S}^{(N+1)} > 0$ and nonsingular $\mathbf{E}^{(0)}, \mathbf{E}^{(1)}, \ldots, \mathbf{E}^{(N+1)}$ satisfying

$$\begin{bmatrix} \left(\mathbf{E}^{(i)}\right)^{\mathrm{T}} + \mathbf{E}^{(i)} - \mathbf{S}^{(i)} & \mathbf{Z}^{(i)}(r) \\ \mathbf{Z}^{(i)}(r)^{\mathrm{T}} & \bar{\mathbf{S}} \end{bmatrix} > 0 , \quad r = 1, \ldots, L ,$$

for $i = 0, 1, \ldots, N+1$, where

$$\mathbf{Z}^{(i)}(r) = \left(\tilde{\mathbf{A}}^{(i)} \mathbf{E}^{(i)}\right)^{\mathrm{T}} \left(\underline{t}^{(i)}(r) \otimes \mathbf{I}_s\right) ,$$

$$\underline{t}^{(i)}(r) = \left[\sqrt{t_{i0}(r)} \ \sqrt{t_{i1}(r)} \ \ldots \ \sqrt{t_{i(N+1)}(r)}\right] , \quad (19)$$

$$\bar{\mathbf{S}} = \mathrm{diag}(\mathbf{S}^{(0)}, \mathbf{S}^{(1)}, \ldots, \mathbf{S}^{(N+1)}) . \quad (20)$$

Proof. The result is proven using arguments similar to those employed to prove Proposition 2 in Aberkane (2011).

Theorem 10 enables us to determine the existence of a stabilizing mode-independent state feedback law

$$\underline{U}_k = \mathbf{L} \underline{\psi}_k , \quad (21)$$

as indicated by the corollary below.

Corollary 11. The polytopic MJLS (5) is mean square stabilizable by mode-independent state feedback (21) if there exist $\mathbf{S}^{(0)}, \mathbf{S}^{(1)}, \ldots, \mathbf{S}^{(N+1)} > 0$ and $\mathbf{E}, \mathbf{K}$ such that for each $i = 0, 1, \ldots, N+1$ the LMIs

$$\begin{bmatrix} \mathbf{E}^{\mathrm{T}} + \mathbf{E} - \mathbf{S}^{(i)} & \tilde{\mathbf{Z}}^{(i)}(r) \\ \tilde{\mathbf{Z}}^{(i)}(r)^{\mathrm{T}} & \bar{\mathbf{S}} \end{bmatrix} > 0 , \quad r = 1, \ldots, L ,$$

with

$$\tilde{\mathbf{Z}}^{(i)}(r) = \left(\left(\tilde{\mathbf{A}}^{(i)} \mathbf{E}\right)^{\mathrm{T}} + \left(\tilde{\mathbf{B}}^{(i)} \mathbf{K}\right)^{\mathrm{T}}\right) \left(\underline{t}^{(i)}(r) \otimes \mathbf{I}_s\right) ,$$

and $\underline{t}^{(i)}(r)$ and $\bar{\mathbf{S}}$ given by (19), (20), are feasible. The stabilizing controller gain is then given by $\mathbf{L} = \mathbf{K}\mathbf{E}^{-1}$.

Proof. The corollary is shown using the same reasoning as in the proof of Theorem 10 by considering the closed-loop dynamics

$$\underline{\psi}_{k+1} = \left(\tilde{\mathbf{A}}^{(\theta_k)} + \tilde{\mathbf{B}}^{(\theta_k)} \mathbf{L}\right) \underline{\psi}_k + \underline{\tilde{w}}_k ,$$

and the change of variables $\mathbf{K} = \mathbf{L}\mathbf{E}$.

An implementation of the proposed control law that uses yalmip (Löfberg, 2004) and SDPT3 (Tütüncü et al., 2003) to evaluate the LMI condition given in Corollary 11 is part of CoCPN-Sim and can be found on github (Jung and Rosenthal, 2018).

5. ILLUSTRATIVE EXAMPLE

In this section, we provide a numerical example to illustrate the derived results by means of a simulation. To that end, we consider a double integrator plant with state $\underline{x}_k = [s_k \ \dot{s}_k]^{\mathrm{T}}$, where s_k denotes the horizontal displacement of the body in meters, and parameters

$$\mathbf{A} = \begin{bmatrix} 1 & t_{\mathrm{a}} \\ 0 & 1 \end{bmatrix} , \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} , \quad \mathbf{W} = 0.1 \begin{bmatrix} t_{\mathrm{a}}^2/2 \\ t_{\mathrm{a}} \end{bmatrix} \begin{bmatrix} t_{\mathrm{a}}^2/2 \\ t_{\mathrm{a}} \end{bmatrix}^{\mathrm{T}} ,$$

in (1), where $t_{\mathrm{a}} = 0.01\,\mathrm{s}$, $m = 2\,\mathrm{kg}$, and the noise is Gaussian distributed.

The goal is to synthesize a stabilizing sequence-based controller that communicates the control inputs to the plant over a network with time-varying packet delay and loss probabilities. For controller synthesis, we use $N = 3$, which corresponds to control sequences of length four, so that the resulting augmented dynamics (5) is a polytopic MJLS with five modes and the transition matrix $\mathbf{P}_k$ varies within a polytope with $L = 192$ vertices. Accordingly, the set $\mathcal{A}_L$ contains 192 matrices, each of which being of dimension 320×320. Using the JSR toolbox from Vankeerberghen et al. (2014), we verify that $1 \leq \rho(\mathcal{A}_L) \leq 1.2447$, implying that the uncontrolled augmented dynamics is not mean square stable. This is expected since the plant dynamics taken by itself is already not (asymptotically) stable.

However, assuming that sequences of length four are large enough to satisfy Assumption 1 and setting $\delta = 0.1$ so that $p_{k,44}$ is assumed to lie within the interval $[0, 0.1]$, we can invoke Corollary 11 to compute a stabilizing state feedback controller with gain

$$\mathbf{L} = \begin{bmatrix} -0.510 & -1.512 & 0.044 & -0.255 & -0.350 & 0.072 & -0.235 & 0.068 \\ -0.085 & -0.199 & -0.029 & 0.009 & -0.009 & -0.026 & 0.004 & -0.020 \\ -0.091 & -0.219 & -0.015 & -0.034 & -0.002 & -0.035 & -0.039 & -0.036 \\ -0.090 & -0.221 & -0.015 & -0.040 & -0.054 & -0.036 & -0.043 & -0.036 \end{bmatrix} .$$

To illustrate that the controller does indeed stabilize the plant, we carry out 1000 simulation runs, each of which is comprised of 10000 time steps. In each run, the initial plant state is randomly drawn from a Gaussian distribution with mean and covariance

$$\underline{\hat{x}}_0 = \begin{bmatrix} 5 \\ 0 \end{bmatrix} , \quad \mathbf{X}_0 = 0.5^2 \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} .$$

To simulate a network with time-varying delay and loss probabilities, at each time step a pmf is chosen by randomly picking a row from the stochastic matrix

$$\begin{bmatrix} 0.0001 & 0.455 & 0.430 & 0.045 & 0.03 & 0.0200 & 0.0199 \\ 0.0001 & 0.455 & 0.400 & 0.045 & 0.02 & 0.0199 & 0.0300 \\ 0.0001 & 0.455 & 0.430 & 0.045 & 0.03 & 0.0199 & 0.0200 \\ 0.0001 & 0.455 & 0.430 & 0.045 & 0.02 & 0.0300 & 0.0199 \\ 0.0001 & 0.455 & 0.045 & 0.430 & 0.02 & 0.0300 & 0.0199 \\ 0.0001 & 0.045 & 0.455 & 0.430 & 0.02 & 0.0300 & 0.0199 \\ 0.0001 & 0.430 & 0.045 & 0.455 & 0.02 & 0.0199 & 0.0300 \\ 0.0001 & 0.045 & 0.430 & 0.455 & 0.02 & 0.0199 & 0.0300 \\ 0.0001 & 0.455 & 0.430 & 0.030 & 0.02 & 0.0199 & 0.0450 \\ 0.0001 & 0.455 & 0.030 & 0.430 & 0.02 & 0.0199 & 0.0450 \end{bmatrix} ,$$

according to which the actual packet delay is then drawn. In each pmf the last entry subsumes the probability that a packet is delayed by more than five time steps or gets lost (infinite delay), whereas the first entry, indicating the probability that a packet is to be delivered without delay, is chosen very small to reflect the behavior of real networks.

After the simulation runs, an estimate of $\mathrm{E}\{\underline{x}_k\}$ is calculated in terms of the sample mean $\underline{\bar{x}}_k$. The evolution of the norm of $\underline{\bar{x}}_k$ over time is depicted in Figure 2 and an exemplary state trajectory from a single run is shown

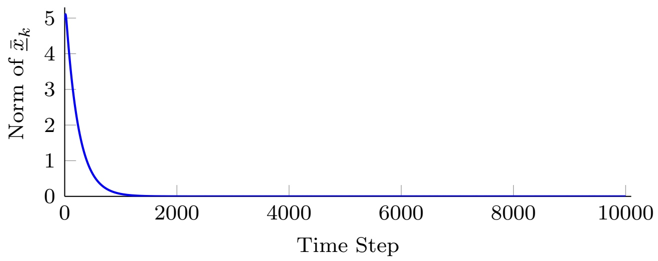

Fig. 2. Norm the sample mean $\bar{x}_k$ of the plant state over time.

in Figure 3. The results indicate that the closed loop system is mean square stable. In fact, they even suggest exponential mean square stability.

6. CONCLUSION

In this work, sequence-based control over networks with time-varying and generally unknown delay and loss probabilities was considered. Stability analysis and controller synthesis were addressed based on a problem reformulation in terms of a polytopic MJLS. Subsequently, a necessary and sufficient condition for mean square stability and a sufficient condition for the existence of a stabilizing state feedback controller were derived and illustrated by means of simulations.

Future research will investigate whether mean square stability and exponential stability, which are known to be equivalent characterizations of noise-free polytopic MJLS, are also equivalent for the considered polytopic MJLS driven by wide-sense stationary noise. While this is strongly suggested by our simulation results, a formal verification is demanded. Additionally, future research will extend the presented results by considering output feedback in setups where sensor data communicated to the controller are also subject to delays and losses.

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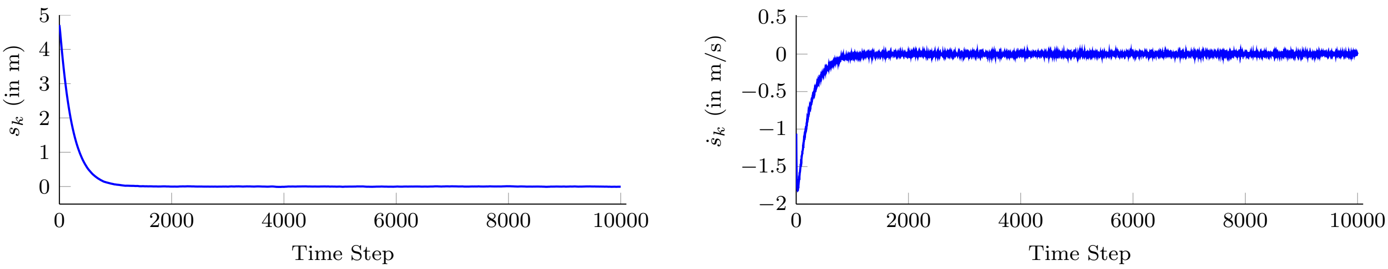

Fig. 3. Evolution of the plant states s_k (left) and $\dot{s}_k$ (right) in one of the simulation runs.

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