Recursive Nonlinear Set–Theoretic Estimation Based on Pseudo Ellipsoids

Uwe D. Hanebeck
Institute of Automatic Control Engineering
Technische Universität München
80290 München, Germany
Uwe.Hanebeck@ieee.org

Abstract

In this paper, the problem of estimating a vector $\underline{x}$ of unknown quantities based on a set of measurements depending nonlinearly on $\underline{x}$ is considered. The measurements are assumed to be taken sequentially and are corrupted by unknown but bounded uncertainties. For this uncertainty model, a systematic design approach is introduced, which yields closed–form expressions for the desired nonlinear estimates. The estimates are recursively calculated and provide solution sets $\mathcal{X}$ containing the feasible sets, i.e., the sets of all $\underline{x}$ consistent with all the measurements available and their associated bounds. The sets $\mathcal{X}$ are tight upper bounds for the exact feasible sets and are in general not convex and not connected. The proposed design approach is versatile and the resulting nonlinear filter algorithms are both easy to implement and efficient.

1 Introduction

Estimating the state of a dynamic system based on a sequence of uncertain measurements is a standard problem in many applications. Usually, this problem is approached in a stochastic setting. Alternatively, set–theoretic methods can be used by assuming a priori bounds on the uncertainties. Estimation then consists of constructing sets of possible states, which are consistent with the a priori bounds and the measurements. Good overviews about this topic can be found in [1, 3, 19].

Most work has been done in set–theoretic state estimation for linear systems [17, 18, 21, 22]. Applications in the field of speech processing are found in [4, 5]. Robotic applications are discussed in [7, 16]. A comparison of stochastic and set–theoretic estimation is given in [6].

In the case of nonlinear systems, the complex sets resulting from the estimation procedure are either approximated by simple–shaped sets, e.g. ellipsoids, boxes [15, 12], polytopes [20], or by the union of simple sets [11, 12].

In [20], an approach similar to the Extended Kalman Filter (EKF) is pursued. As in the EKF, the nonlinear state equations are linearized about the current state estimate. Unlike the EKF, linearization errors are not neglected, but rather considered as additional exogeneous disturbances. Estimation is performed recursively and provides polytopes as approximation of the posterior feasible set.

The procedure in [15] works without linearization of the nonlinear state equations and provides (recursively) the smallest axis–aligned box enclosing the posterior feasible set.

In [11], the posterior feasible set is characterized by enclosing it between internal and external unions of boxes on the basis of interval analysis. Recursive estimation has not been addressed. A more advanced version of this approach with lower computational complexity has been introduced in [12].

In this paper, a new nonlinear filtering algorithm for nonlinear systems is proposed, that does not rely in any way on linearization. In addition, the new approach is not based on a grid or on propagating particles, but provides a finite–dimensional closed–form representation of the resulting complex–shaped sets. This includes nonconvex sets or sets that are not even connected. When applying the new filter recursively to a sequential stream of measurements, the size of the analytical representation of the resulting sets does not grow with the number of measurements.

The key idea of the proposed filter is to transform the original N–dimensional space S to an L–dimensional hyperspace S^* with $L > N$. This results in an N–dimensional manifold U^*, called the universal manifold, in the L–dimensional transformed space S^*. Complex–shaped subsets of the original N–dimensional space are then represented by N–dimensional submanifolds of U^* in the space S^*.

These submanifolds are defined by the intersection of L–dimensional simple–shaped sets, e.g. ellipsoids, with the universal manifold U^*. Furthermore, the nonlinear measurement equation is transformed to a linear one in the hyperspace S^*. Hence, nonlinear filtering can be performed by a linear filter operating in the transformed space S^*.

Section 2 formulates the problem of nonlinear set–theoretic estimation. In Sec. 3 the concept of modeling complex–shaped sets is introduced. The nonlinear filtering algorithm is given in Sec. 4. The performance of the new nonlinear filtering algorithm is demonstrated by estimating the parameters of a SCARA–type robot manipulator in Sec. 5.

2 Problem Formulation

Consider a nonlinear discrete–time dynamic system with system state $\underline{x}_k$ (not directly observable) at time step k. Measurements $\hat{\underline{z}}_k$ of the system output are taken at time instants $k = 1, 2, \ldots$ according to the nonlinear measurement equation

$$\hat{\underline{z}}_k = \underline{h}_k(\underline{x}_k) + \underline{v}_k \tag{1}$$

with measurement uncertainty $\underline{v}_k$, which represents exogenous noise sources or model parameter uncertainties.

The uncertainties $\underline{v}_k$, $k = 1, 2, \ldots$, are assumed to be bounded by a known set $\mathcal{V}_k$ according to $\underline{v}_k \in \mathcal{V}_k$. The set can be of complicated shape, i.e., can be nonconvex or not connected.

The goal is to estimate at each time instant k the state $\underline{x}_k$ based on *all* available measurements $\hat{\underline{z}}_l$ for $l = 1, 2, \ldots, k$. Of course, a recursive estimation procedure is preferred, which calculates a state estimate based on the estimate at the previous time step and the current measurement. Therefore, it is not required to store and reprocess all measurements. Furthermore, instead of trying to construct point estimates, we prefer to calculate at each time instant k all states that are compatible with the measurements and their corresponding uncertainties.

On a theoretical level, the problem can easily be solved: Let $\mathcal{X}^s_{k-1}$ denote the set of all states compatible with all the measurements up to time step $k-1$ and their respective uncertainties. Furthermore, $\mathcal{X}^m_k$ denotes the set of states defined solely by the measurement *at* time k according to

$$\mathcal{X}^m_k = \{\underline{x}_k : \hat{\underline{z}}_k - \underline{h}_k(\underline{x}_k) \in \mathcal{V}_k\} \quad .$$

Then the estimate $\mathcal{X}^s_k$ is given by the intersection

$$\mathcal{X}^s_k = \mathcal{X}^s_{k-1} \cap \mathcal{X}^m_k \quad .$$

However, representing these sets in practical applications at least approximately by a finite set of parameters is not a trivial task. On one hand, the parameter set should not be too large, even more, the approximation should degrade gracefully with a decreasing number of parameters. On the other hand, the number of parameters should not be permanently growing with an increasing number of incoming measurements. Hence, the remainder of this paper is concerned with a new parametric representation of complex–shaped sets and an efficient procedure for calculating the corresponding parameters.

3 Pseudo Ellipsoids

The key idea of this paper is to represent an uncertainty $\mathcal{X}_k$ with a complicated shape in the N–dimensional original space S by a simpler shaped uncertainty $\mathcal{X}^*_k$ in an L–dimensional hyperspace S^* with $L > N$. Points $\underline{x}_k$ in S are related to points $\underline{x}^*_k$ in S^* via a nonlinear transformation $\underline{T}(.)$ according to

$$\underline{x}^*_k = \underline{T}(\underline{x}_k) = \begin{bmatrix} T_1(\underline{x}), & T_2(\underline{x}), & \ldots, & T_L(\underline{x}) \end{bmatrix}^T \quad .$$

Hence, $\underline{T}(.)$ defines an N–dimensional manifold U^* in an L–dimensional space.

In addition, L–dimensional sets $\mathcal{X}^*_k$ of simple shape are defined in the transformed space S^*. Here, ellipsoidal sets according to

$$\mathcal{X}^*_k = \left\{ \underline{x}^*_k : (\underline{x}^*_k - \hat{\underline{x}}^*_k)^T (\mathbf{X}^*_k)^{-1} (\underline{x}^*_k - \hat{\underline{x}}^*_k) \leq 1 \right\}$$

are used, where $\hat{\underline{x}}^*_k$ is the ellipsoid midpoint and $\mathbf{X}^*_k$ is a symmetric positive definite matrix.

The intersection of an ellipsoid $\mathcal{X}^*_k$ with the manifold U^* defines a submanifold of U^*, which, in turn, defines a complicated set in the original space S.

Remark 3.1 A complex–shaped set in the original space S is defined by *both* the transformation $\underline{T}(.)$ *and* the pseudo ellipsoid $\mathcal{X}^*_k$.

In many cases, the type of transformation $\underline{T}(.)$ results directly from the nonlinearities considered. For example, when considering polynomial nonlinearities, a polynomial transformation is used. For trigonometric nonlinearities, a trigonometric transformation can be used.

However, to simplify application of the new filtering approach, a generic transformation is helpful. For that purpose, Bernstein polynomials are used, since their approximation capabilities are sufficient for a large class of nonlinearities. In addition, they lead to better conditioned calculations than standard polynomials. Multidimensional Bernstein polynomials are defined on the basis of one–dimensional Bernstein polynomials, which on the interval $[l, r]$ are given by

$$H_i^n(x) = \left(\begin{array}{c} n \\ i \end{array} \right) \left(\frac{l-x}{l-r} \right)^i \left(\frac{r-x}{r-l} \right)^{n-i}$$

for $i = 0, \ldots, n$. With

$$\underline{x}_k = \begin{bmatrix} x_{k,1} & x_{k,2} & \ldots & x_{k,N} \end{bmatrix}^T \quad ,$$

the above transformation is defined by

$$T_i(\underline{x}_k) = \prod_{j=1}^{N} H_{i_j}^{L_j - 1}(x_{k,j}) \quad ,$$

for $i_j = 0, \ldots, L_j - 1$, $j = 1, \ldots, N$, $L = \prod_{j=1}^{N} L_j$, and $i = \sum_{j=1}^{N} i_j$.

4 Filtering

Based on the concept of pseudo ellipsoids, which represent complex–shaped sets in the original space S by pseudo ellipsoids in the hyperspace S^*, the nonlinear filter step can now be performed by a linear filter in the hyperspace S^*. For that purpose, a pseudo–linear expansion of the nonlinear measurement equation $\underline{h}_k(.)$ is performed according to

$$\underline{h}_k(\underline{x}_k) = \mathbf{H}_k^* \underline{x}_k^* + \underline{e}_k^h \approx \mathbf{H}_k^* \underline{x}_k^* \quad ,$$

where $\underline{e}_k^h$ represents the approximation error defined by $\underline{e}_k^h = \underline{h}_k(\underline{x}_k) - \mathbf{H}_k^* \underline{x}_k^*$.

In general, the expansion can be enhanced by an additional nonlinear transformation $\underline{g}(.)$ of the measurements according to

$$\underline{g}(\hat{\underline{z}}_k - \underline{v}_k) = \underline{g}(\underline{h}_k(\underline{x}_k)) \quad .$$

The left hand side can be approximated by

$$\underline{g}(\hat{\underline{z}}_k - \underline{v}_k) = \hat{\underline{z}}_k^* - \mathbf{G}_k^* \underline{v}_k^* - \underline{e}_k^{v,*} \approx \hat{\underline{z}}_k^* - \mathbf{G}_k^* \underline{v}_k^* \quad ,$$

where $\hat{\underline{z}}_k^*$ and $\mathbf{G}_k^*$ are nonlinear functions of $\hat{\underline{z}}_k$ and $\underline{v}_k^*$ is a nonlinear function of $\underline{v}_k$. $\underline{e}_k^{v,*}$ accounts for the approximation error.

The right hand side is again approximated according to

$$\underline{g}(\underline{h}_k(\underline{x}_k)) = \mathbf{H}_k^* \underline{x}_k^* + \underline{e}_k^{h,*} \approx \mathbf{H}_k^* \underline{x}_k^*$$

with approximation error $\underline{e}_k^{h,*}$. As a result, the measurement equation in the hyperspace is obtained according to

$$\underline{z}_k^* = \mathbf{H}_k^* \underline{x}_k^* + \underbrace{\underline{e}_k^{h,*} + \mathbf{G}_k^* \underline{v}_k^* + \underline{e}_k^{v,*}}_{\underline{w}_k^*}$$

with $\underline{w}_k^*$ representing the total uncertainty.

Let the set of all predicted states be given by the set $\mathcal{X}_k^p$, which is defined in the transformed space S^* by

$$\mathcal{X}_k^{p,*} = \left\{ \underline{x}_k^* : (\underline{x}_k^* - \hat{\underline{x}}_k^{p,*})^T (\mathbf{E}_k^{p,*})^{-1} (\underline{x}_k^* - \hat{\underline{x}}_k^{p,*}) \leq 1 \right\} \quad .$$

Furthermore, let $\underline{w}_k^*$ be bounded by the set

$$\mathcal{W}_k^* = \left\{ \underline{w}_k^* : (\underline{w}_k^*)^T (\mathbf{W}_k^*)^{-1} \underline{w}_k^* \leq 1 \right\} \quad .$$

Then, the set defined by the measurement is given by

$$\mathcal{X}_k^{m,*} = \Big\{ \underline{x}_k^* : (\hat{\underline{z}}_k^* - \mathbf{H}_k^* \underline{x}_k^*)^T (\mathbf{W}_k^*)^{-1} (\hat{\underline{z}}_k^* - \mathbf{H}_k^* \underline{x}_k^*) \leq 1 \Big\} \quad .$$

The fusion result is given by a set $\mathcal{X}_k^{s,*}$ (again an ellipsoid in the transformed space, but a set of complicated shape in the original space!) that contains the intersection of the ellipsoids $\mathcal{X}_k^{p,*}$ and $\mathcal{X}_k^{m,*}$. Hence, $\mathcal{X}_k^{s,*}$ is obtained by a linear set–theoretic filter in the hyperspace S^* [19]

$$\mathcal{X}_k^{s,*} = \left\{ \underline{x}_k^* : (\underline{x}_k^* - \hat{\underline{x}}_k^{s,*})^T (\mathbf{E}_k^{s,*})^{-1} (\underline{x}_k^* - \hat{\underline{x}}_k^{s,*}) \leq 1 \right\}$$

with

$$\hat{\underline{x}}_k^{s,*} = \hat{\underline{x}}_k^{p,*} + \lambda_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \left\{ \mathbf{W}_k^* + \lambda_k^* \mathbf{H}_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \right\}^{-1} \left(\hat{\underline{z}}_k^* - \mathbf{H}_k^* \hat{\underline{x}}_k^{p,*} \right) \quad ,$$

and

$$\begin{aligned} \mathbf{E}_k^{s,*} &= d_k^* \, \mathbf{P}_k^{s,*} \\ \mathbf{P}_k^{s,*} &= \mathbf{E}_k^{p,*} - \lambda_k^* \, \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \left\{ \mathbf{W}_k^* + \lambda_k^* \, \mathbf{H}_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \right\}^{-1} \mathbf{H}_k^* \mathbf{E}_k^{p,*} \quad , \end{aligned}$$

where

$$d_k^* = 1 + \lambda_k^* - \lambda_k^* \left(\hat{\underline{z}}_k^* - \mathbf{H}_k^* \hat{\underline{x}}_k^{p,*} \right)^T \left\{ \mathbf{W}_k^* + \lambda_k^* \, \mathbf{H}_k^* \mathbf{E}_k^{p,*} (\mathbf{H}_k^*)^T \right\}^{-1} \left(\hat{\underline{z}}_k^* - \mathbf{H}_k^* \hat{\underline{x}}_k^{p,*} \right) \quad .$$

Using this form of bounding ellipsoid for the exact intersection of $\mathcal{X}_k^{p,*}$, $\mathcal{X}_k^{m,*}$ in the transformed space S^* offers the advantage that the resulting set $\mathcal{X}_k^s(\lambda_k)$ *in the original space* S possesses the following property, which is desirable in applications in the sense, that no new uncertainty is introduced:

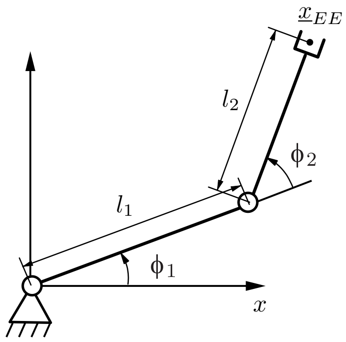

Figure 1: Schematical top view of the considered type of 2D SCARA robot manipulator with segment lengths l_1, l_2 and joint angles ϕ_1, ϕ_2.

LEMMA 4.1 *If the two sets $\mathcal{X}_k^p$ und $\mathcal{X}_k^m$ overlap, the filtering result $\mathcal{X}_k^s(\lambda_k)$ contains the exact intersection $\mathcal{X}_k^p \cap \mathcal{X}_k^m$ and is itself contained in their union $\mathcal{X}_k^p \cup \mathcal{X}_k^m$. Hence, it holds*

$$(\mathcal{X}_k^p \cap \mathcal{X}_k^m) \subset \mathcal{X}_k^s(\lambda_k) \subset (\mathcal{X}_k^p \cup \mathcal{X}_k^m)$$

for all $\lambda_k \in [0, \infty]$.

The fusion parameter λ_k is selected in such a way, that a certain measure of the size of the set $\mathcal{X}_k^s$ is minimized. (How to obtain the minimum volume ellipsoid in a linear setting is discussed in [2].)

5 Simulation Example

We consider a SCARA–type robot manipulator with two degrees of freedom according to Fig. 1. The segment lengths l_1, l_2 are assumed to be unknown and are not amenable to direct measurements. Only uncertain measurements of the distance r of the end–effector from the origin are available for different angles ϕ_1, ϕ_2. The position of the end–effector with respect to the origin is given by

$$\underline{x}_{EE} = \begin{bmatrix} l_1 \cos(\phi_1) + l_2 \cos(\phi_1 + \phi_2) \\ l_1 \sin(\phi_1) + l_2 \sin(\phi_1 + \phi_2) \end{bmatrix} \ .$$

Hence, the distance r of the end–effector from the origin is given by

$$r = \sqrt{l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\phi_2)} \ .$$

Given this nonlinear relation, the segment lengths l_1, l_2 are estimated ($N = 2$) based on measured distances $\hat{r}_k$, $k = 1, 2, \ldots$ for different angles $\phi_{2,k}$. The measurement uncertainties are assumed to be bounded according to

$$|r_k - \hat{r}_k| \leq R \ ,$$

which gives

$$\hat{r}_k = \sqrt{l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\phi_{2,k})} + v_k \text{ with } v_k^2 \leq R^2 \ .$$

The parameters used during the simulation are

$$\begin{aligned} l_1 &= 500 \text{ mm} \ , \\ l_2 &= 350 \text{ mm} \ , \\ R &= 40 \text{ mm} \ . \end{aligned}$$

The filtering procedure starts with an axis–aligned uncertainty box

$$\mathcal{X}_1^p = [0, 1000] \times [0, 1000] \text{ mm}^2 \ .$$

Sequentially, measurements $\hat{r}_k$, $k = 1, \ldots, 4$, for $\phi_{2,1} = 90^o$, $\phi_{2,2} = 120^o$, $\phi_{2,3} = 160^o$, $\phi_{2,4} = 170^o$ are used to update the initial estimate.

The function $\underline{g}(.)$ has been selected to $\underline{g}(x) = \begin{bmatrix} x^2 & x^4 & x^6 \end{bmatrix}^T$, $T_i(\underline{x})$, $i = 1, \ldots, L$, are chosen as multidimensional Bernstein polynomials. The results are visualized[1] for $n = 6$ in Fig. 2, where n is the order of the one–dimensional Bernstein polynomials, which gives $L = 49$. Obviously, the result of the proposed new filter is a tight approximation of the exact estimation result at every time step.

6 Conclusions

In this paper, an efficient algorithm for the recursive calculation of tight closed–form approximations of the feasible sets in nonlinear set–theoretic estimation problems has been presented. The resulting sets are a much better approximation compared to simple sets like hyper–rectangles or ellipsoids and are in general of complex shape, i.e., nonconvex and not connected.

The same methodology has been applied to *stochastic* nonlinear systems [10]. For the case of mixed stochastic and set–theoretic uncertainties, the filtering algorithms presented in [8, 9] will be generalized to nonlinear systems.

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[1] The result of updating the estimate based on measurement k is denoted $\mathcal{X}_k^s$. However, in the next time step, the same estimate is denoted $\mathcal{X}_{k+1}^p$ to be consistent with the notation introduced above.

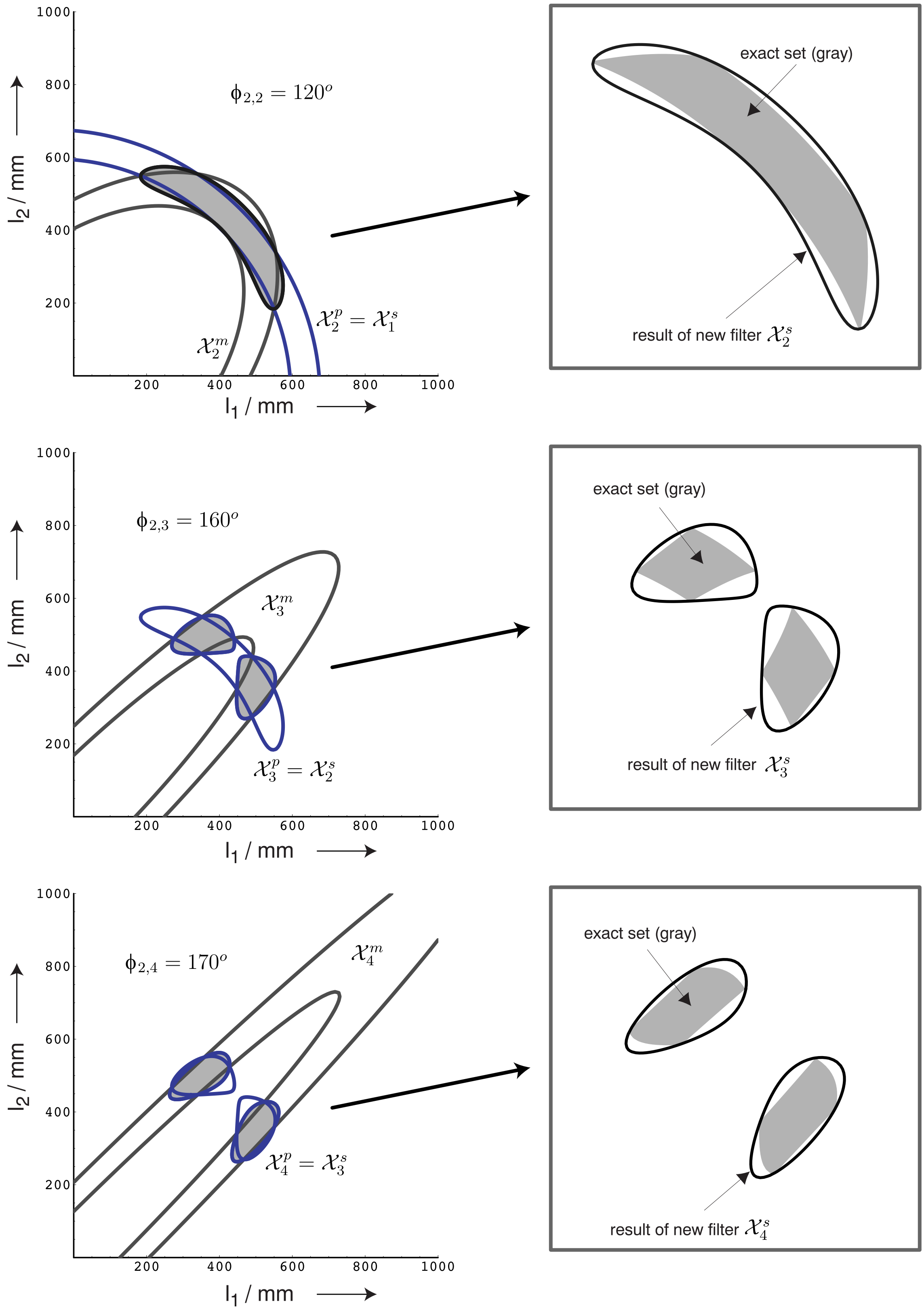

Figure 2: Results of sequential estimation of the lengths l_1, l_2 of the 2D SCARA robot manipulator.

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