Probabilistic instantaneous model-based signal processing applied to localization and tracking

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A B S T R A C T

In this paper, a probabilistic approach for estimating time and space-variant parameters of a system, based on sequentially received discrete-time signal values, is presented. The system description is the solution of a linear partial differential equation (PDE). The PDE describes for example the wave propagation of an acoustic wave in a localization system. The solution of the PDE is given by a time-variant and space-variant impulse response. This impulse response is characterized by the time and space-variant parameters in order to track an object, which emits for example an acoustic signal. For estimating the position of the object in an instantaneous way a Bayesian approach has to be used, which considers the dynamic behavior of the parameters in a system model and uncertainties in a stochastic manner by means of probability density functions. Hence, the new approach provides a probabilistic instantaneous model-based signal processing, where the sequentially measured signal values are processed directly and known reference signal sequences are interpreted as part of a time-variant nonlinear measurement equation.

1. Introduction

1.1. Motivation

Model-based signal processing gives the possibility to consider the physical behavior of the observed phenomena. In doing so the desired filtered information has a higher signal-to-noise ratio as methods, where the physical behavior is not explicitly considered [7]. In a localization system for example, several sources emit different signals, which are received by several sinks attached to an object (Fig. 1). Based on the emitted and the received signals, it is possible to estimate the position and the orientation of the object. For localization and tracking, standard approaches first estimate the time-of-flight, based on the signals emitted by several sources and received by several sinks, which is then subsequently used by a range-based localization approach to estimate the target's position [3,21].

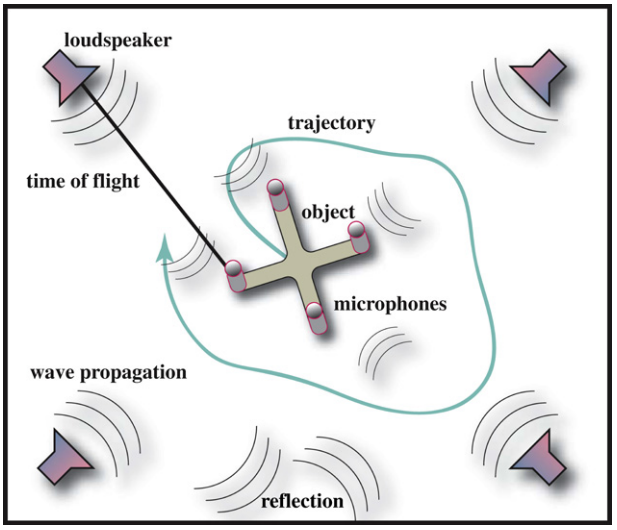


Fig. 1. Example of an acoustic tracking system.

1.2. State-of-the-art

Standard approaches for estimating the times-of-flight assume that the desired parameters are stationary and deterministic. These algorithms neglect uncertainties inherent in the signal and do not explicitly consider the time-variant nature of the signal parameters. Matched filters [10] are optimal regarding the maximum likelihood criteria, if the noise is additive, zero-mean, and Gaussian distributed. The noise can be white or colored [18]. The matched filter can be interpreted as a kind of cross-correlation between the emitted signal and the received signal. Reflections and linear distortions of the emitted signal are not considered in

the matched filter. Hence, in [23,22] an extended matched filter is introduced. To handle overlapping echoes, a parallel bank of different matched filters is used, where the transmitted signal is modeled as a non-stationary auto covariance function. Similar to this approach in [24,5] the stochastic matched filter is presented. These kinds of filter are block-wise algorithms, where in the case of a continuous signal, the processing is done by means of consecutive blocks of samples. There, the trade-off between maximizing signal-to-noise ratio and minimizing peak smearing, which occurs in the case of moving sink or source, has to be considered. This depends on the block length [21].

Other algorithms for time-of-flight estimation are adaptive filters. They adjust the filter coefficients in such a way that the difference between the measured and the filtered samples is minimized. This allows one to consider slowly time-varying parameters. However, the adaptation parameter of the adaptive filter has to be set in such a way that the algorithm does not become unstable.

In [1,2] Bayesian state estimators (Extended Kalman Filter, Unscented Kalman Filter) are used for estimating the time-of-flight, where the signal envelope is regarded as a measurement equation.

As stated above, standard approaches estimate the position of a source based on the times-of-flight. The estimation can be achieved by range-based localization approaches [16,9,8]. They use a geometry-based approach [16], closed-form solutions [9], solutions of the nonlinear measurement equation by means of Taylor-series expansions [8], gradient descent procedures or nonlinear filter approaches such as the extended Kalman filter.

An extension of the matched filter is the matched field processing algorithm. These algorithms model the acoustic field based on the wave propagation and use the signals to estimate the positions in a model-based fashion [7,20,6]. In contrast to the matched filter the matched field processing algorithm compare the received signals with the expected emitted signals at every possible position. To consider linear distortion of the signal, in [12, 11] a modified Green function, which modeled the channel, is used. This leads to a matched field processor that is adjusted to the signal and to the channel.

Because the matched field processor modeled the whole acoustic field, a mismatch problem occurs [20]. In this case the model does not fit the true environment, because no uncertainties for the model are assumed. To overcome this problem a probabilistic state estimator can be used. In [19] an extended Kalman filter is applied for estimating the position and the signal parameters.

1.3. Key ideas of this paper

In this article, estimating unknown time and space-variant parameters from a set of discrete-time signal sequences of a partially unknown system is considered. The proposed approach will be exemplified by two scenarios, namely time-of-flight estimation and localization. In both scenarios, the desired parameters are estimated based on time series.

In general, the relationship between the desired parameters and the received signals is described by a nonlinear measurement model. In addition, the object typically has a nonlinear dynamic behavior, which results in a nonlinear system model. To handle these nonlinearities and to consider model uncertainties and noise, a Bayesian estimator has to be used that can cope with nonlinearities.

The probabilistic instantaneous model-based signal processing approach introduced in this paper employs a model of wave propagation, which leads to a time-variant and space-variant impulse response. The impulse response is converted into a state space model. The resulting state vector can be infinite-dimensional. A dead time, which correspond in localization to the time-of-arrival, would be in a state space model infinite-dimensional and uncountably infinite. Based on a appropriate transformation this is avoidable. Hence, in the proposed approach known reference sequences are interpreted as part of a nonlinear measurement equation, which converts the time variables on amplitude values. Furthermore, a system model for the dynamic behavior of the desired parameters and its associated uncertainty are incorporated. Based on a probabilistic system description, which includes the measurement and the system equation, a Bayesian estimator is used. This allows to estimate the parameters from time sequences directly, e.g., the time-of-flight or the position of a source. Therefore, uncertainties are explicitly considered and a probability density function over the desired parameter space is determined.

The new approach immediately processes every given signal sample in a recursive fashion. In contrast, conventional methods, e.g. matched filter, estimate the time-of-flight from overlapping or consecutive blocks of samples, if the continuous emitted signal is split into small parts. Hence, the computational complexity of the proposed new approach is much lower, because all previous measurements are represented in the probability density function and only the current measurement is processed. In addition, the smearing effect is reduced due to the provided system model for the desired parameters.

Based on a generative measurement model and reasonable assumptions, a reduced measurement equation can be derived. The reduced measurement equation is used to estimate the desired parameter by using a Bayesian state estimator. In doing so, the probabilistic instantaneous model-based signal processing approach was presented and exemplified for time-of-flight estimation in [4]. For this example, the implementation of the Bayesian state estimator is based on a representation of the probability density functions by means of Gaussian mixtures. In order to use Gaussian mixtures for representing the likelihood function, the sample values of the known source signal are linearly interpolated and used as part of a time-variant measurement equation. The number of mixture components is limited, because the time update is performed based on an approximation of the underlying transition density by means of a Gaussian mixture with axis-aligned Gaussian components. By doing so, only the weighting factors of the components have to be updated. The approach has been extended to localization, where a nonlinear state estimator has to be used to cope with nonlinearities. In this paper, a hybrid density filter for estimating the position of the source is applied.

1.4. Structure of the paper

The structure of this paper is as follows. A problem formulation for estimating the parameters relating to time signals is presented in Section 2. The general procedure for the probabilistic instantaneous model-based signal processing approach is introduced in Section 3. The approach is exemplified by two scenarios, namely time-of-flight estimation and localization. In order to derive the reduced measurement equation, necessary assumptions are discussed in Section 4.1. For time-of-flight estimation, the measurement model and the system equation is determined in Section 4.2. Furthermore, the Bayesian state estimator used is presented. The measurement model for localization and a short overview of the Hybrid Density Filter (HDF) used for position estimation is given in Sections 4.3 and 4.3.1, respectively. The performance of the new approach compared with the traditional solution is evaluated in Section 5. Simulation and experimental results are also shown. Conclusions are given in Section 6.

2. Problem formulation

The wave equation for the propagation of small-amplitude acoustic signals in an ideal, homogeneous medium can be described by a linear partial differential equation

$$\nabla^2 y\left(t, \underline{x}\right) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} y\left(t, \underline{x}\right) = s\left(t, \underline{x}\right), \tag{1}$$

where $y\left(t, \underline{x}\right)$ is the sound pressure at location $\underline{x}$ at the time t, $s\left(t, \underline{x}\right)$ denotes the input signals in the medium by several sources located in space, and c is the velocity of sound. The solution of (1) can be described as a linear, causal, time-variant, and space-variant impulse response [25]. The impulse response $h\left(t, \underline{x}, \tau, \underline{x}_0\right)$ of this time-variant and space-variant system can be described as the response at time t and at position $\underline{x}$ of a Dirac delta distribution emitted at time τ and at position $\underline{x}_0$. While the time-variant part of the impulse response describes moving objects in the environment, e.g., the sources or the sensors, the physical topology of the environment, e.g., scatter objects or reflections, are represented by the space-variant part. Thus, the output signal $y\left(t, \underline{x}\right)$ of the linear, causal, time-variant, and space-variant system is given by the integral

$$y\left(t, \underline{x}\right) = \int_{\mathbb{R}^3}\int_0^t s\left(\tau, \underline{x}_0\right) h\left(t, \underline{x}, \tau, \underline{x}_0\right) \mathrm{d}\tau \mathrm{d}\underline{x}_0 + v\left(t\right), \tag{2}$$

where additionally, the filtered signal is corrupted by additive noise $v\left(t\right)$. Furthermore, the noise process, also referred to as measurement noise, includes uncertainties arising from modeling errors. If several moving point sources located at $\underline{x}_s^j\left(t\right)$ are considered, the input signals in the medium are given by

$$s\left(t, \underline{x}\right) = \sum_{j=1}^{N} {}_{h_I}\, s_d^j\left(t\right) \delta\left(\underline{x} - \underline{x}_s^j\left(t\right)\right), \tag{3}$$

where ${}_{h_I}$ is an interpolation function and $s_d^j\left(t\right)$ is a discrete-time signal according to the source j. Inserting (3) into (2) and utilizing the properties of the Dirac delta distribution (2) can be simplified to

$$y\left(t, \underline{x}\right) = \sum_{j=1}^{N}\int_0^t {}_{h_I}\, s_d^j\left(\tau\right) h\left(t, \underline{x}, \tau, \underline{x}_s^j\left(\tau\right)\right) \mathrm{d}\tau + v(t).$$

We consider a certain sensor i with position $\underline{m}_i\left(t\right)$ that receives the signal $y\left(t, \underline{x} = \underline{m}_i\left(t\right)\right)$. Sampling the received signal at sensor i leads to an amplitude measurement

$$\begin{aligned} {}_{y_l^i} &:= y\left(lT, \underline{m}_i\left(lT\right)\right) \\ &= \sum_{j=1}^{N}\int_0^{lT} {}_{h_I}\, s_d^j\left(\tau\right) h\left(lT, \underline{m}_i\left(lT\right), \tau, \underline{x}_s^j\left(\tau\right)\right) \mathrm{d}\tau + v\left(lT\right) \end{aligned} \tag{4}$$

for each discrete-time step lT, where T is the sampling interval. From these sequentially received amplitude measurements, a parameter vector $\underline{p}_l$ that describes the model of the time-variant and space-variant system (4) has to be estimated. These parameters include, for example time-of-flight, which emerges from the distance between a microphone and a loudspeaker or the position of a loudspeaker, if several microphones receive the signal. For M sensors and the parameter vector $\underline{p}_l$, the *generative measurement equation* is given by

$$\underbrace{\begin{bmatrix} y_l^1 \\ \vdots \\ y_l^M \end{bmatrix}}_{\underline{y}_l} = \underline{g}_l\left(\underline{p}_l\right) + \underbrace{\begin{bmatrix} v_l^1 \\ \vdots \\ v_l^M \end{bmatrix}}_{\underline{v}_l}, \tag{5}$$

where

$$\underline{g}_l\left(\underline{p}_l\right) = \begin{bmatrix} \sum_{j=1}^{N}\int_0^{lT} {}_{h_I}\, s_d^j\left(\tau\right) h\left(lT, \underline{m}_1\left(lT\right), \tau, \underline{x}_s^j\left(\tau\right)\right) \mathrm{d}\tau \\ \vdots \\ \sum_{j=1}^{N}\int_0^{lT} {}_{h_I}\, s_d^j\left(\tau\right) h\left(lT, \underline{m}_M\left(lT\right), \tau, \underline{x}_s^j\left(\tau\right)\right) \mathrm{d}\tau \end{bmatrix}.$$

Eq. (5) describes the relationship between the measured values and the emitted time signals subject to known and unknown parameters. A major novelty is the interpretation of known reference time signals as part of a nonlinear measurement equation. The impulse response in (5) has to be converted into a state space model. Later in this paper a non-dispersive medium is assumed. In the case of a dispersive medium a modified Green function [11] can be converted to a state space model, where the parameters of the model can either be known or unknown.

Example 1. This example shows that under some assumptions this approach is equivalent to the matched filter. First a free-field model is assumed and the parameter is deterministic and stationary. In the case, if the desired parameter is the time-of-flight p, (5) is given by

$$\begin{aligned} {}_{y_l} &= \int_0^{lT} {}_{h_I}\left(s_d\left(\tau\right)\right) \delta\left(lT - \tau - p\right) \mathrm{d}\tau + v_l \\ {}_{y_l} &= {}_{h_I}\left(s_d\left(lT - p\right)\right) + v_l. \end{aligned}$$

If a discrete-time channel is considered and the parameter p is a multiple of the sampling interval, the mapping function based on the emitted discrete-time signal is ${}_{s_{lT-p}}$. Because the parameter is stationary, the measurement vector contains of all measurements from time step 0 to k. For estimating the parameter, a maximum likelihood estimator is applied. The noise is assumed as additive, zero-mean, and Gaussian distributed. If only the cross-effect between the measurement vector and the emitted signal, which depends on the parameter, is considered and the noise is white, the estimate for the parameter p is given by

$$\hat{p} = \arg\max_p \sum_{k=0}^{K} {}_{y_k}\, {}_{s_{k-p}}.$$

Example 2. In this example the parameter has a dynamic behavior. The dynamic is described by a differential equation

$$p' = V, \tag{6}$$

where V is the velocity of the parameter and p' the first time derivative. If a free-field model is assumed and the emitted signal is a sine-function, then the measurement equation is given by

$${}_{y_l} = \sin\left(2\pi f\left(lT - p\left(lT\right)\right)\right) + v_l. \tag{7}$$

Solving (6) leads to

$$p\left(t\right) = V \cdot t + p\left(0\right) \tag{8}$$

(8) is used in (7), which results in

$$\begin{aligned} {}_{y_l} &= \sin\left(2\pi f\left(lT - p\left(lT\right)\right)\right) + v_l \\ {}_{y_l} &= \sin\left(2\pi f l T \underbrace{\left(1 - V\right)}_{\text{Time stretch factor}} - 2\pi f p\left(0\right)\right) + v_l. \end{aligned}$$

The stretch factor is a time scaling factor, which occurs when the object is moving. This effect can be modeled based on a system equation (6). In the next section, a stochastic differential equation (9), which describes the dynamic behavior, is used.

The two examples show that the new approach leads to the conventional method, if the parameter is assumed to be deterministic and stationary. Furthermore, if a system model for the parameter is applied, the compression effect is considered.

3. General approach

The general approach, proposed here, for estimating unknown time-variant and space-variant parameters of a system is based on time sequences, the generative measurement equation (5) and a stochastic system model. However, solving the integral in (5) is computationally demanding or impossible in general. For a large number of real-world applications the generality of (5) is not required. Thus, simplifying (5) by incorporatory application specific assumptions will lead to a reduced measurement equation. This reduced measurement equation offers the possibility for easily estimating the parameter vector $\underline{p}_l$ from the time signals $s_d^j(t)$ and the measurement vector $\underline{y}_l$. The derivation and necessary assumptions are described in Section 4.1. Subject to a specific problem, like time-of-flight estimation, additional simplifications can be considered.

In addition to the reduced measurement equation a system equation has to be identified. The system equation describes the dynamic behavior of the parameters, which can be written as

$$\underline{p}_{l+1} = \underline{a}_l(\underline{p}_l) + \underline{w}_l, \tag{9}$$

where $\underline{a}_l(\cdot)$ is a nonlinear mapping and $\underline{w}_l$ is the additive process noise. In the system equation the dynamic of the parameter is considered.

Due to the nonlinear measurement and nonlinear system equation affected by additive noise, a Bayesian state estimator is used for estimating the desired parameters. The generic Bayesian estimator consists of two parts, the measurement update and the time update. For several amplitude measurements $\underline{y}_l$ at time lT the measurement update is given by

$$f_l^e(\underline{p}_l) = c f_l^L(\underline{y}_l|\underline{p}_l) f_l^p(\underline{p}_l), \tag{10}$$

where $f_l^e(\underline{p}_l)$ is the estimated density function and c a normalization constant. The likelihood function $f_l^L(\underline{y}_l|\underline{p}_l)$ depends on (5). The predicted density function $f_l^p(\underline{p}_l)$ represents prior knowledge about the estimated parameters. In the time update, a new predicted density is calculated according to

$$f_l^p(\underline{p}_{l+1}) = \int_{\mathbb{R}} f^T(\underline{p}_{l+1}|\underline{p}_l) \cdot f_l^e(\underline{p}_l)\, \mathrm{d}\underline{p}_l,$$

where $f_l^e(\underline{p}_l)$ is the estimated density from the measurement update (10) and $f^T(\underline{p}_{l+1}|\underline{p}_l)$ is the transition density, which depends on the system equation (9).

4. The probabilistic instantaneous model-based signal processing

Based on the generative measurement equation (5) and the general approach, the proposed model-based signal processing approach will be exemplified for two scenarios, namely time-of-flight estimation and localization.

4.1. Assumptions

In order to solve the integral (5), some assumptions have to be made. However, the presented general approach is not restricted to these assumptions.

4.1.1. Assumption I

First, only one moving point source located at $\underline{x}_s(t)$ is considered and the emitted signal $s_d(t)$ is known. Second, the sensors are assumed to be static. A third assumption is, that some unknown parameters are bounded and only a finite time window for the integration needs to be considered. Based on these assumptions the measurement equation for a certain sensor i is given by

$$y_l^i = \int_{D_{\lim}T}^{lT} h_I(s_d(\tau))\, h(lT, \underline{m}_i, \tau, \underline{x}_s(\tau))\, \mathrm{d}\tau + v_l^i. \tag{11}$$

4.1.2. Assumption II

If we *assume* that the system is time-variant and space-invariant, then the system describes a free-field model

$$h(t, \tau, \underline{m}_i - \underline{x}_s(\tau)) = h(t-\tau, t)$$

$$h(t-\tau, t) = a(\underline{m}_i, \underline{x}_s(t))\, \delta(t - \tau - f(\underline{m}_i, \underline{x}_s(t), c)). \tag{12}$$

In the case of a moving source the time-of-flight is varying. Hence, the system depends on the time-varying distance between the source and the sensor position. A further *assumption* is that the attenuation term $a(\underline{m}_i, \underline{x}_s(t))$ can be neglected. Thus, using (11) in (11) results in the *reduced measurement equation*

$$y_l^i = h_I\left(\sum_{k=l-D_{\lim}}^{l} c_k \delta(lT - kT - f(\underline{m}_i, \underline{x}_s(lT), c))\right) + v_l^i, \tag{13}$$

where c_k are the sample values of the emitted signal.

While assumption I is made only for brevity and clarity, assumption II is very strict. In order to use a more realistic model, the unknown parameter vector has to be extended. In doing so, the parameter vector includes the attenuation term, reflections and for example the velocity of sound. For instance, reflections can be modeled by virtual sources, where the parameters are described by the attenuation factors and the boundaries.

4.2. Example I: Time-of-flight estimation

In this example a time-shift between two discrete-time series is estimated. To achieve an efficient probabilistic algorithm the known reference signal $s_d(t)$ is linearly interpolated between the sample values c_k and c_{k+1}. The resulting continuous-time signal $s(t)$, which is interpreted as a nonlinear time-variant measurement equation, is given by

$$s(t) = \sum_k (a_k t + b_k)\, \mathrm{rect}\left(\frac{t}{T} - \frac{1}{2} - k\right). \tag{14}$$

where

$$\mathrm{rect}(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

is the rectangular function and

$$a_k = \frac{c_{k+1} - c_k}{T}, \qquad b_k = (1+k)\, c_k - k\, c_{k+1}.$$

Based on the linear interpolation function in (14) the reduced measurement equation (13) is given by

$$y_l = \sum_{k=l-D_{\lim}}^{l} (a_k(lT - \tau_l) + b_k)\, \mathrm{rect}\left(\frac{lT - \tau_l}{T} - \frac{1}{2} - k\right) + v_l \tag{15}$$

for one amplitude measurement, where the nonlinear function $f(\cdot)$ is substituted by the time-of-flight τ_l

$$f(\underline{m}, \underline{x}_s(lT), c) = \tau_l.$$

In analogy to the measurement equation, a system equation, which is given in (9), has to be identified. In this paper we consider a simple linear model with additive time invariant process noise w_l according to

$$\tau_{l+1} = \tau_l + w_l.$$

Thus, the uncertainty of the parameter increases over time.

4.2.1. Derivation of a probabilistic model

In order to derive the likelihood function according to the measurement equation, we use the model given in (15) and can thus define the likelihood function f^L in the additive noise case as

$$f_l^L(y_l|\tau_l) = f^v\left(y_l - \sum_{k=l-D_{\lim}}^{l} (a_k(lT-\tau_l)+b_k) \times \operatorname{rect}\left(\frac{lT-\tau_l}{T} - \frac{1}{2} - k\right)\right).$$

We assume the density f^v to be Gaussian, resulting in

$$f_l^L(y_l|\tau_l) = \mathcal{N}\left(y_l - \sum_{k=l-D_{\lim}}^{l} (a_k(lT-\tau_l)+b_k) \times \operatorname{rect}\left(\frac{lT-\tau_l}{T} - \frac{1}{2} - k, \sigma_v\right)\right).$$

This function is piecewise Gaussian, as the rectangular functions do not overlap. Hence, the sum and the rectangular functions can be extract from the exponent, so that the likelihood function can be simplified to

$$f_l^L(y_l|\tau_l) = \sum_{k=l-D_{\lim}}^{l} \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left(-0.5\left(\frac{\tau_l-\mu_k}{\sigma_k}\right)^2\right) \times \operatorname{rect}\left(\frac{\tau_l}{T} + 0.5 + k - l\right), \tag{16}$$

where

$$\mu_k = \frac{-y_l + a_k lT + b_k}{a_k}, \qquad \sigma_k = \frac{\sigma_v}{a_k}.$$

For efficient implementation, the rectangular function can be approximated by a Gaussian mixture according to

$$\operatorname{rect}\left(\frac{\tau_l}{T} + 0.5 + k - l\right) = \sum_{j=1}^{M} \omega_j \mathcal{N}\left(\tau_l - \mu_k^j, \sigma^j\right),$$

with

$$\omega_j = \frac{T}{M}, \qquad \mu_k^j = (l-k-1)T + (j-0.5)\frac{T}{M} \quad \text{and}$$
$$\sigma^j = \sigma\frac{T}{M},$$

because the Gaussian mixture representation possesses the universal approximation property [17]. This approximation of the rectangular function is substituted in (16). The resulting expression is a multiplication of Gaussian mixtures

$$f_l^L(y_l|\tau_l) = \sum_{k=l-D_{\lim}}^{l} \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left(-0.5\left(\frac{\tau_l-\mu_k}{\sigma_k}\right)^2\right) \sum_{j=1}^{M} \omega_j \mathcal{N}\left(\tau_l - \mu_k^j, \sigma^j\right)$$
$$= \sum_{k=l-D_{\lim}}^{l} \sum_{j=1}^{M} \underbrace{\omega_k}_{\frac{\sigma_k}{\sigma_v}} \mathcal{N}(\tau_l - \mu_k, \sigma_k) \omega_j \mathcal{N}\left(\tau_l - \mu_k^j, \sigma^j\right).$$

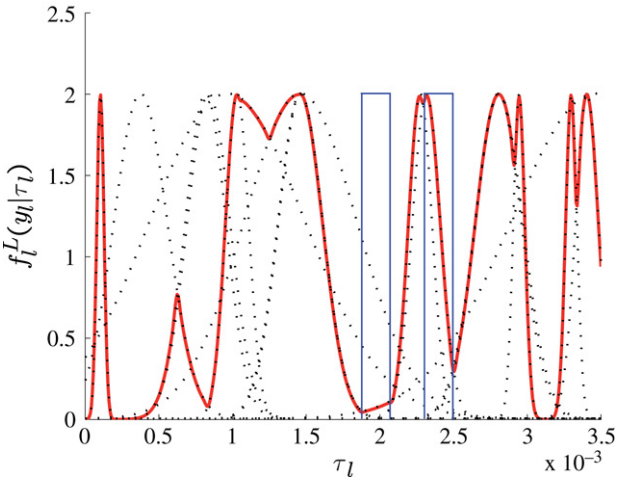


Fig. 2. Likelihood function for a certain measurement.

The multiplication result can be combined to a Gaussian mixture with $M \cdot (D_{\lim}+1)$ components. The resulting likelihood function is given by

$$f_l^L(y_l|\tau_l) = \sum_{u=0}^{M\cdot D_{\lim}} \omega_u^g \mathcal{N}\left(\tau_l - \mu_u^g, \sigma_u^g\right),$$

where the components are calculated by the following expressions

$$\mu_u^g = \frac{\mu_k\sigma^{j2} + \mu_k^j\sigma_k^2}{\sigma^{j2}+\sigma_k^2}, \qquad \sigma_u^g = \sqrt{\frac{\sigma^{j2}\sigma_k^2}{\sigma^{j2}+\sigma_k^2}},$$
$$\omega_u^g = \frac{\omega_k\omega_j}{\sqrt{2\pi\left(\sigma^{j2}+\sigma_k^2\right)}} \exp\left(-\frac{1}{2}\frac{\left(\mu_k-\mu_k^j\right)^2}{\sigma^{j2}+\sigma_k^2}\right), \tag{17}$$

with $k = l - D_{\lim}, \ldots, l$ and $j = 1, \ldots, M$. Fig. 2 shows the likelihood function for a certain measurement.

4.2.2. Measurement update

In the measurement update, the estimated density $f_l^e(\tau_l)$ can be calculated according to Bayes' law

$$f_l^e(\tau_l) = c f_l^L(y_l|\tau_l) f_l^p(\tau_l).$$

As stated above, the approximated likelihood function consists of $M \cdot (D_{\lim}+1)$ Gaussian mixture components and the predicted density $f_l^p(\tau_l)$ is a Gaussian mixture with N components. The estimated density $f_l^e(\tau_l)$ has $N \cdot (M \cdot (D_{\lim}+1))$ Gaussian mixture components. For calculating the components of the estimated density (17) can be used. To obtain a valid density, the estimated density has to be normalized.

4.2.3. Time update

According to the given system equation, the transition density is given by

$$f^T(\tau_{l+1}|\tau_l) = \int_{\mathbb{R}} \delta(w - \tau_{l+1} + \tau_l) f^w(w)\,\mathrm{d}w = f^w(\tau_{l+1} - \tau_l).$$

The process noise f^w is assumed to be Gaussian and the transition density is approximated with an axis-aligned Gaussian mixture with N components

$$f^T(\tau_{l+1}|\tau_l) = \sum_{i=1}^{N} \omega_i \mathcal{N}(\tau_{l+1} - \mu_i, \sigma_i)\,\mathcal{N}(\tau_l - \mu_i, \sigma_i).$$

In order to use a more realistic process noise model, the density can be approximated by a Gaussian mixture. In this case, the number of the components of the resulting transition density is increasing, depending on the approximation of the process noise. If a nonlinear transition density model is used, it can be approximated with the algorithm described in [13]. By using this transition density, the predicted density

$$f_l^p(\tau_{l+1}) = \int_{\mathbb{R}} \sum_{i=1}^{N} \omega_i \mathcal{N}(\tau_{l+1} - \mu_i, \sigma_i)\, \mathcal{N}(\tau_l - \mu_i, \sigma_i)\, f_l^e(\tau_l)\, \mathrm{d}\tau_l$$

can be written as

$$f_l^p(\tau_{l+1}) = \sum_{i=1}^{N} \mathcal{N}(\tau_{l+1} - \mu_i, \sigma_i) \underbrace{\omega_i \int_{\mathbb{R}} \mathcal{N}(\tau_l - \mu_i, \sigma_i)\, f_l^e(\tau_l)\, \mathrm{d}\tau_l}_{\omega_i^p}.$$

That means, that only the weighting factors of the components for τ_{l+1} have to be updated. The resulting predicted density has N components.

4.3. Example II: Localization

The other example considered in this paper is localization. Instead of estimating the time-of-flight between a source and a sensor, the position of the source is directly estimated based on the received amplitude measurement values of M sensors by using the probabilistic instantaneous model-based signal processing approach. Instead of using a linear interpolation, a more realistic function $h_I(\cdot)$ is used. The sinc-function $\mathrm{si}(x) = \frac{\sin(\pi x)}{\pi x}$ is an ideal interpolator for band-limited continuous time signals [10]. This interpolation function is used in (13), resulting in a time-variant measurement equation for a certain sensor i

$$y_l^i = \sum_{k=l-D_{\mathrm{lim}}}^{l} c_k \mathrm{si}\left(\frac{l-k}{T} - \underbrace{\frac{\|\underline{m}_i - \underline{x}_l\|}{c}}_{=f(\underline{m}_i, \underline{x}_s(lT), c)}\right) + v_l^i$$

where $\underline{x}_l$ is the parameter, which has to be estimated. The resulting measurement equation for the M sensors is given in vector-matrix-notation by

$$\underline{y}_l = \mathbf{g}(\underline{x}_l, \underline{H}_l) + \underline{v}_l, \tag{18}$$

where

$$\underline{y}_l = \begin{bmatrix} y_l^1 \\ \vdots \\ y_l^M \end{bmatrix}, \quad \underline{H}_l = \begin{bmatrix} c_{l-D_{\mathrm{lim}}} \\ \vdots \\ c_l \end{bmatrix}, \quad \underline{v}_l = \begin{bmatrix} v_l^1 \\ \vdots \\ v_l^M \end{bmatrix},$$

$$\mathbf{g}(\underline{x}, \underline{H}_l) = \begin{bmatrix} \mathrm{si}\left(\frac{D_{\mathrm{lim}}}{T} - \frac{\|\underline{m}_1 - \underline{x}_l\|}{c}\right) & \cdots & \mathrm{si}\left(-\frac{\|\underline{m}_1 - \underline{x}_l\|}{c}\right) \\ \vdots & \vdots & \vdots \\ \mathrm{si}\left(\frac{D_{\mathrm{lim}}}{T} - \frac{\|\underline{m}_M - \underline{x}_l\|}{c}\right) & \cdots & \mathrm{si}\left(-\frac{\|\underline{m}_M - \underline{x}_l\|}{c}\right) \end{bmatrix}.$$

The system equation is assumed to be a simple linear model described in Section 4.2. Of course, this model can be replaced with a more realistic system model characterizing the motion of the tracked object.

4.3.1. Hybrid density filter (HDF)

The likelihood function is defined for the additive noise case according to the measurement equation in (18) as

$$f_l^L(\underline{y}_l | \underline{x}_l, \underline{H}_l) = f^v(\underline{y}_l - \mathbf{g}(\underline{x}_l, \underline{H}_l)).$$

The noise is assumed to be zero-mean, white, and Gaussian distributed.

In order to estimate the position of a source directly from the measurement according to (18) the Hybrid Density Filter (HDF) is used. This Bayesian state estimator provides the opportunity of accurate estimation in combination with an efficient processing due to its novel approximation technique [15, 14]. The key idea of the HDF is to approximate the transition density and the likelihood function, which is the probabilistic representation of the nonlinear system and the measurement model, with a hybrid density. This hybrid density is a mixture density consisting of Dirac components and Gaussian components. Instead of a separate measurement update step, a combined time and measurement update is proposed to decrease the computational load [14]. Thus, it is sufficient to approximate the transition density only. Performing the time update then additionally updates the system state by means of the current measurement.

5. Results

The performance of the proposed framework is evaluated in simulation as well as in experiments with real data. Furthermore, the new algorithm is compared with conventional methods, the matched filter [18] and a range-based localization approach [3].

5.1. Time-of-flight estimation

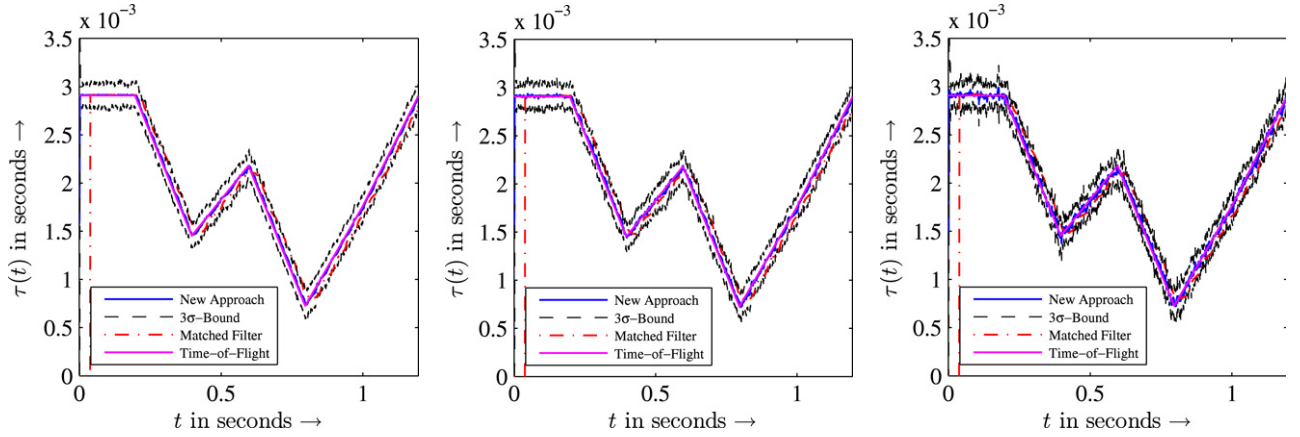
5.1.1. Simulation results

In the simulation setup, a moving source is simulated with a piecewise constant velocity. The source signal is delayed depending on the time-variant distance between sensor and source. The emitted signal is a realization of a zero-mean white Gaussian process with unit variance. In addition, the emitted signal is corrupted by additive zero-mean white Gaussian noise with different variances. Eq. (13) is used to generate the measurement samples, where a sinc-function for the interpolation is used. The nonlinear function $f(\cdot)$ is given by

$$f(\underline{m}, \underline{x}(t), c) = \frac{\|\underline{m} - \underline{x}(t)\|}{c}.$$

The sampling frequency is set to $f_t = 4800$ Hz. For the new approach, the parameters for the likelihood function and for the transition density were selected as follows. For the likelihood function, the maximum time-of-flight is $\tau_{\max} = 0.0035$ s. The measurement noise is set to $\sigma_v = 1$. Each rectangular function is approximated by $M = 10$ components and the variance is set to $\sigma = 0.7$. Each approximated rectangular function overlaps, which results in an error for the likelihood function. The quadratic error for the approximation of one rectangular function is 0.0296%. Each variance of the likelihood function depends on the time-variant amplitude values. The variance decreases if the distance between the amplitude values increases. Obviously, the variance depends on the frequency spectrum of the signal. The transition density is approximated with $N = 60$ components. The expected values of the components are equally spaced points between $\mu_{tr} = [0, 0.0035]$ s. The variance of the transition density is set to $\sigma_{tr} = 1.52 \times 10^{-5}$. It has to be adjusted with respect to the sampling frequency, because in the time update only the weighting factors of the predicted density are updated.

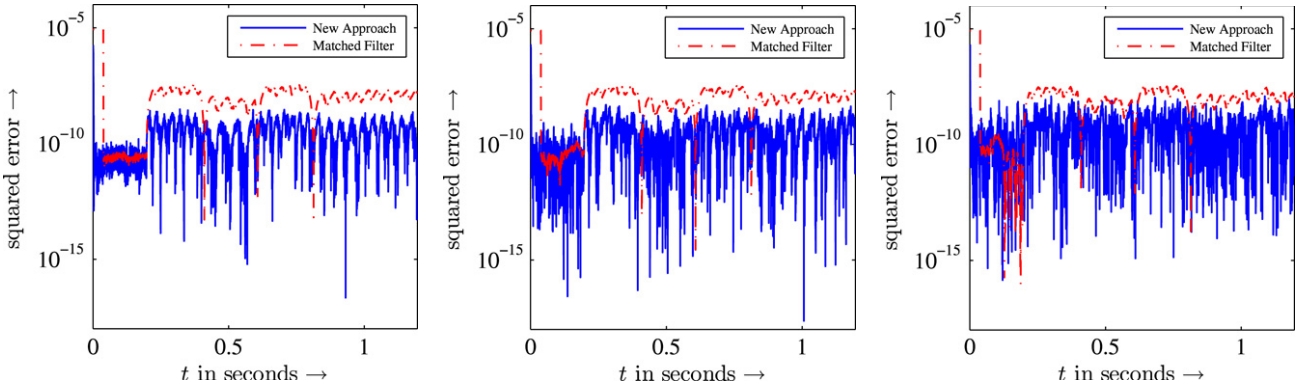
In the matched filter the block length is set to 200 samples, which corresponds to 41.6 ms. The matched filter is calculated



(a) Comparison of the new algorithm and the conventional method in the simulation with a noise level $\sigma_v = 0$.

(b) Comparison of the new algorithm and the conventional method in the simulation with a noise level $\sigma_v = 0.5$.

(c) Comparison of the new algorithm and the conventional method in the simulation with a noise level $\sigma_v = 1$.



(d) Squared error at noise level $\sigma_v = 0$.

(e) Squared error at noise level $\sigma_v = 0.5$.

(f) Squared error at noise level $\sigma_v = 1$.

Fig. 3. Simulation result for a moving source. The matched filter is compared with the new probabilistic approach. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

for every sample, which means that the block is shifted when a sample is received. Based on the results of the matched filter, the maximum value is selected then. To achieve a subsample resolution a quadratic interpolation is used.

In the simulations, the variance of the noise term is set to 0, 0.5 and 1, respectively. The results are shown in Fig. 3, where the new approach (blue line), the matched filter (red line) and the true time-of-flight (magenta line) are illustrated. The new approach has a transition time of 14 samples, where the matched filter provides a result after 200 samples. Furthermore, the matched filter assumes that the parameter is constant during the data block. Therefore a smearing effect occurs. In the new approach, this effect does not occur, because by using a system model the estimator is not too confident and so a measurement can influence the estimate. Furthermore, prior information about the parameters are represented in the predicted density, which is used for estimation. The squared error between the true time-of-flight and the estimates is illustrated in Fig. 3. The squared error of the estimates from the new approach is smaller than from the matched filter. Even if the noise level is very high, the squared error of the estimates from the new approach is lower than 10^{-8}.

5.1.2. Experimental results

In the experimental setup, a source moves for three seconds towards the sensor and then back to the starting point. The maximum distance deviation is 0.34 m. The velocity of sound is assumed to be 343 m/s, thus the distance deviation is

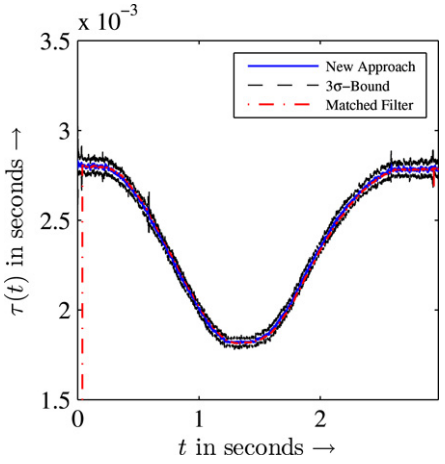
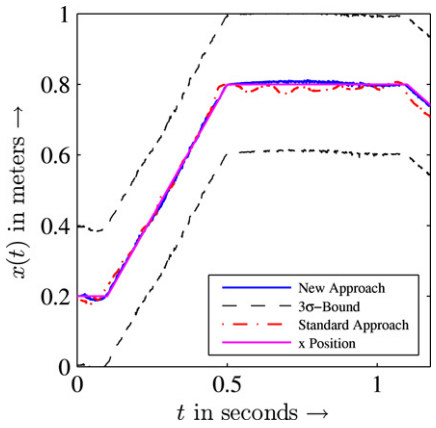
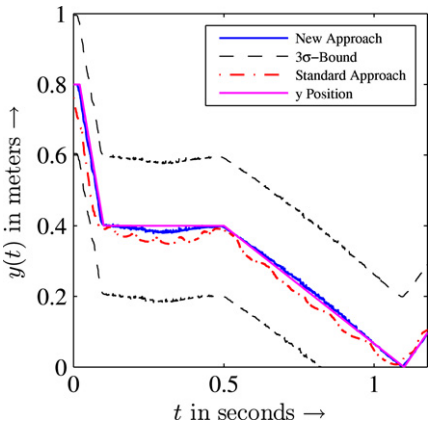


Fig. 4. Experimental result for a moving source, comparing classic and new approach. The sampling frequency is set to $f_t = 48000$ Hz.

0.99125×10^{-3} s. In Fig. 4, the results are shown. The sampling frequency is 48000 Hz. The parameters for the new approach are as follows. The maximum time-of-flight of the likelihood function is $\tau_{\max} = 0.0035$ s. The measurement noise is set to $\sigma_v = 0.5$. Each rectangular function is approximated by $M = 10$ components and the variance is set to $\sigma = 0.7$. The transition density is approximated by $N = 200$ components. The expected values are



(a) Comparison of the new approach and the standard approach in x direction without noise.



(b) Comparison of the new approach and the standard approach in y direction without noise.

Fig. 5. Simulation results for the x and y coordinates of the trajectory.

equally spaced points between $\mu_{tr} = [0.0015, 0.0035]$ s. The variance of the transition density is set to $\sigma_{tr} = 2.12 \times 10^{-6}$.

The block length for the matched filter is set to 2000. The new approach provides results with high accuracy similarly as the matched filter. Even so, the estimates of the matched filter can produce outliers, because no previous information is used.

To reduce the complexity of the new approach, the signals can be downsampled first. In the next experiment, the two time signals are downsampled with a factor of 10 resulting in a sampling frequency of 4800 Hz. If the data is downsampled, the parameter for the transition density has to be adjusted. The transition density can be approximated with a lower number of components, here $N = 96$. The expected values are equally spaced points between $\mu_{tr} = [0.0015, 0.0035]$ s and the variance of the transition density is set to $\sigma_{tr} = 5.2 \times 10^{-6}$.

The block length for the matched filter is set to 200 with respect to the new sampling frequency (red line). In doing so the matched filter produced outliers. When the block length is increased to 500 (green line), the estimate of the standard approach is much more accurate. The results are shown in Fig. 6. The accuracy is lower in comparison with the higher sampling frequency. The linear interpolation does not interpolate the underlying continuous signal very accurate. In that case, a sinc-function interpolation has to be used in order to achieve a much more accurate result.

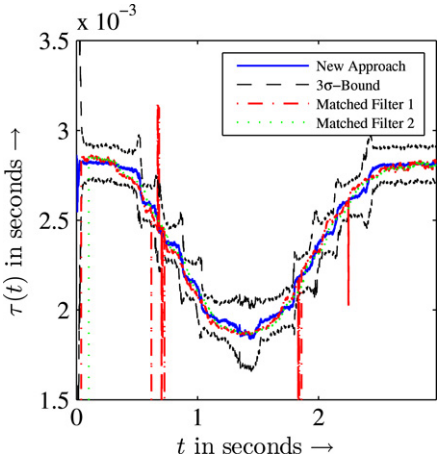


Fig. 6. Experimental result for a moving source, comparing classic and new approach. The sampling frequency is set to $f_t = 4800$ Hz. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

5.2. Localization

5.2.1. Simulation results

The simulation setup is similar as described in the first part of Section 5.1.1. Now, the emitted signal is received by four sensors. The new approach is compared with the standard approach. The standard approach uses a matched filter for calculating the four times-of-flight, where the block length of the matched filter is set to 200. To achieve a subsample resolution, each of the four results is interpolated by a quadratic function. Based on these four times-of-flight estimates, the position of the source is calculated based on a closed-form solution [3]. In Figs. 5 and 7 the estimated x and y trajectory for the cases without noise and with noise ($\sigma_v = 0.5$) are shown.

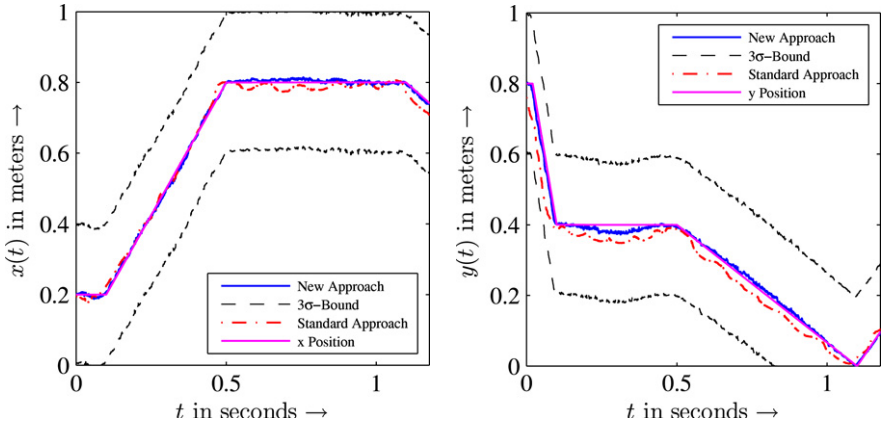
Using the standard approach, the effect of peak smearing appears. The matched filter assumes that the system is time-invariant and then no motion is assumed. This results in estimates being either below or above the ground truth. Furthermore, the angles of intersections of the circles depend also on the position, whereas small angles of intersection are much more sensitive as angles about 90 degree. In contrast, the new approach provides estimates with a smaller error as the standard approach, even if the noise level is increased, because for estimating the position the measurements of all sensors are used together compare with a conventional method, where intermediated states are calculated first. For example, the maximum absolute error is 0.02 m in y direction. This error occurs due to the approximation of the density, the intersection, and the time resolution. This error decreases if the sampling frequency increases.

5.2.2. Experimental results

In the experimental setup, a moving source emits a signal, which is received by four sensors. The sampling frequency is set to 48000 Hz.

In the standard approach, two different algorithms for estimating the times-of-flight are used. The first one is a standard matched filter. The second filter is matched to the signal and to the channel to be more robust to reflections. In a first step, the impulse response of the channel is estimated and then convolved with the emitted signal. The resulting signal is used then in the matched filter. For both matched filters, the block length was 2000.

The estimates are interpolated by a quadratic function resulting in a subsample resolution. Afterwards, a position estimate is determined by using a closed-form range-based localization approach [3].



(a) Comparison of the new approach and the standard approach in x direction with noise.

(b) Comparison of the new approach and the standard approach in y direction with noise.

Fig. 7. Simulation results for the x and y coordinates of the trajectory.

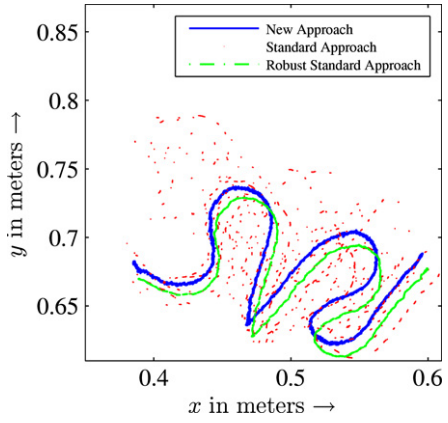


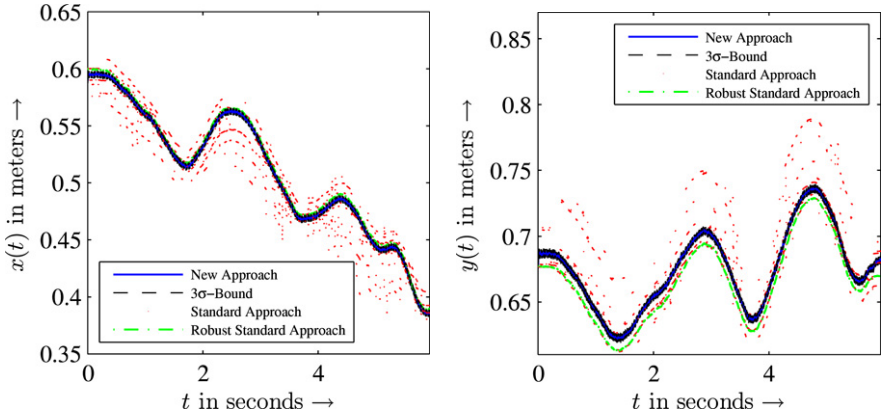
Fig. 8. Experimental result for a moving source. Comparing classic and new approach.

In Fig. 8, the position estimates are illustrated. The new approach can track the trajectory of the source, where the estimates from the standard approach (standard matched filter) are either below or above the trajectory, which is shown in Fig. 9. This is due to distance mismeasurements of the matched filter, because the first reflected path is close to the direct path. The robust matched filter (green line) can track the trajectory, because the influence of the channel is compensated. However, the channel impulse response has to be estimated in a preprocessing step. On the other hand, the new approach provides an integrated solution for this problem, where uncertainties are explicitly considered and consequently a higher accuracy can be provided.

6. Conclusions

An instantaneous model-based signal processing approach for estimating unknown parameters from time sequences has been introduced. The proposed method is exemplified for time-of-flight estimation and source localization. The main contribution of the proposed framework is the end-to-end problem formulation and an integrated solution for estimating parameters from time signals under consideration of model uncertainties and measurement uncertainties, characterized in a stochastic manner. Hence, the new approach provides probability density functions describing the parameter estimates that are updated based on every received measurement sample.

A further novelty provided by the model-based framework is the interpretation of a known reference sequence as part of a nonlinear measurement equation. Furthermore, a system model for the time-varying parameters is included. Compared with conventional methods, the processing delay is reduced, since for every measured value, the probability density function for the desired parameter can be updated. The introduced Bayesian estimators are only an instrument for solving the proposed



(a) Comparison of the new approach and the standard approach in x direction.

(b) Comparison of the new approach and the standard approach in y direction.

Fig. 9. Experimental results for the x and y coordinates of the trajectory. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

problem. Different Bayesian estimators can be easily integrated in order to further improve the accuracy of the parameter estimation.

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