Kalman Filter-based SLAM with Unknown Data Association using Symmetric Measurement Equations

Marcus Baum, Benjamin Noack, and Uwe D. Hanebeck

***Abstract*— This work investigates a novel method for dealing with unknown data associations in Kalman filter-based Simultaneous Localization and Mapping (SLAM) problems. The key idea is to employ the concept of Symmetric Measurement Equations (SMEs) in order to remove the data association uncertainty from the original measurement equation. Based on the resulting modified measurement equation, standard nonlinear Kalman filters can estimate the full joint state vector of the robot and landmarks without explicitly calculating data association hypotheses.**

I. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is a fundamental problem in robotics [6], [10], [30], [31]. The objective of SLAM is to successively localize a robot while simultaneously creating a map of its environment by means of landmark observations. For this purpose, many different approaches have been proposed in literature, e.g., graph-based SLAM such as [3], [4], [7]–[9], [13], [16], [28], [29] and particle filter-based SLAM such as FastSLAM [22], [23]. In this work, we focus on Kalman filter-based approaches, which use, e.g., the Extended Kalman Filter (EKF) [6], [12] or Unscented Kalman Filter (UKF) [21] for recursively calculating the posterior mean and covariance of the full joint state vector of the robot and landmark states.

The standard formulation of Kalman filter-based SLAM assumes that the associations between measurements and landmarks are known, which is rarely the case in real world applications. In this context, several techniques and extensions have been proposed to deal with unknown data associations in SLAM [22]. For example, the Nearest Neighbor (NN) approach [1], [25] assigns the nearest measurement to each landmark with respect to the Mahalanobis distance. More precise methods find joint compatible associations such as in the Joint Compatible Branch and Bound (JCBB) [25] method. These methods have in common that a single data association hypothesis is used to update the joint state vector. If this association is wrong, e.g., due to high noise, the Kalman filter may diverge.

Recently more sophisticated data association techniques from the multi-target tracking community [1] have been used for the purpose of SLAM. For example, the Probability Hypotheses Density (PHD) filter SLAM method [24] is capable of estimating both the number and states of the landmarks. In [5], a SLAM method based on the labeled multi-Bernoulli filter is presented. These approaches have in common that Rao-Blackwellization is performed in order to decouple the robot state from the landmark states and that the correlation between landmarks is not maintained.

In this work, we propose a novel approach for dealing with unknown data associations in Kalman filter-based SLAM. Our method performs an implicit data association, i.e., no data association hypotheses are calculated at all. In contrast to existing implicit association methods for SLAM, it is capable of maintaining the full joint state vector of the robot and the landmarks. For this purpose, we employ the so-called Symmetric Measurement Equation (SME) approach [14], [15], [19], [20], which has been developed for multi-target tracking. The key idea is to rewrite the original measurement equation with a symmetric transformation in order to remove the unknown data association. The modified measurement equation can be fed into any standard nonlinear Kalman filter such as the Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) [11], or S2KF [27]. With a sample-based filter such as the UKF or S2KF, there is no additional computational complexity in comparison to the known data association case (although more samples might lead to improved results due to the introduced nonlinearity).

Note, in this work, we restrict ourselves to a simplified setting: We assume that each landmarks is detected in each scan and that there are no clutter measurements. Extensions of the SME approach to a more general setting are possible [18], but left for future investigations.

The structure of this paper is as follows: First, we introduce the general Kalman filter-based framework for SLAM in Section II and point out the data association problem. Subsequently, in Section III, we show how unknown data associations can be removed from the original SLAM formulation with the help of symmetric transformations constructed from polynomials or kernel functions. By means of a simple SLAM setting in Section IV, we show that SME approach can outperform standard SLAM data association techniques.

II. KALMAN FILTER-BASED SLAM

Kalman filter-based SLAM approaches [6], [12], [21] consider the joint state vector

$$\underline{x}_k := \begin{pmatrix} \underline{x}_k^R \\ \underline{x}_k^{L_1} \\ \vdots \\ \underline{x}_k^{L_m} \end{pmatrix} \quad , \tag{1}$$

where

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- k is the discrete time index,
- $\underline{x}_k^R \in \mathbb{R}^{d_R}$ is the d_R-dimensional robot state,
- m is the number of landmarks, and
- $\underline{x}_k^{L_i} \in \mathbb{R}^{d_L}$, $i \in \{1, \ldots, m\}$, are the d_L-dimensional landmark states.

A. System Model

The temporal evolution of the robot state is specified by a system equation of the form

$$\underline{x}_{k+1}^R = a^R(\underline{x}_k^R, \underline{v}_k^R) \ , \tag{2}$$

where $\underline{v}_k^R$ is zero-mean Gaussian system noise. In the same manner, the system models for the landmarks L_i, $i = 1 \ldots m$, are

$$\underline{x}_{k+1}^L = a^L(\underline{x}_k^L, \underline{v}_k^{L_i})$$

with zero-mean Gaussian noise $\underline{v}_k^{L_i}$. Hence, the overall system model can be written as

$$\underline{x}_{k+1} = a(\underline{x}_k, \underline{v}_k) = \begin{pmatrix} a^R(\underline{x}_k^R, \underline{v}_k^R) \\ a^L(\underline{x}_k^{L_1}, \underline{v}_k^{L_1}) \\ \vdots \\ a^L(\underline{x}_k^{L_m}, \underline{v}_k^{L_m}) \end{pmatrix} \tag{3}$$

with $v_k := \left[(\underline{v}_k^R)^T, (\underline{v}_k^{L_1})^T, \ldots, (\underline{v}_k^{L_m})^T \right]^T$.

B. Measurement Model

For the sake of simplicity, we assume that the robot receives measurements from all m landmarks at each time k, i.e., there are no missing measurements and no background clutter is received.

Hence, given are m d_y-dimensional measurements $\underline{y}_k^1, \ldots, \underline{y}_k^m$ with

$$\underline{y}_k^{\pi_k(i)} = h^i(\underline{x}_k, \underline{w}_k^i) \ , \tag{4}$$

where

- $\pi_k : \{1, \ldots, m\} \to \{1, \ldots, m\}$ is the measurement-to-landmark association,
- $h^i(\underline{x}_k, \underline{w}_k^i)$ relates the robot state to the landmark L_i, and
- $\underline{w}_k^i$ is the measurement noise (for $i = 1, \ldots, m$).

In this work, we consider the problem that the measurement-to-landmark association is unknown π_k.

C. Nonlinear Kalman Filter

Kalman filter-based SLAM aims at recursively calculating the posterior mean $\hat{x}_{k|k}$ and covariance $\mathbf{C}_{k|k}$ for the full joint state vector (1) given all measurements up to time k. For this purpose, alternating prediction and measurement update steps are performed.

In the prediction step, the mean and covariance for the time step $k-1$ are propagated to the next time step k, where $\hat{x}_{k|k-1}$ and $\mathbf{C}_{k|k-1}$ denotes the predicted mean and covariance. In the measurement update step, the prediction for time k is updated based on the received measurements $\underline{y}_k^1, \ldots, \underline{y}_k^m$.

If the data association is known, the prediction and update can be calculated exactly for linear system and measurement functions (3) and (4). In case of nonlinear models, standard nonlinear estimators such as the EKF or the UKF can be employed.

III. Symmetric Measurement Equation Approach for Unknown Data Assocation

In this section, we present a novel approach for Kalman filter-based SLAM in case the data association π_k in (4) is unknown, i.e., it is not known which measurement comes from which landmark. The objective is to formulate an equivalent measurement equation that does not contain the measurement-to-landmark association. This approach resembles the so-called *Symmetric Measurement Equation (SME)* filter [14], [15], which has been originally developed in the context of multi-target tracking. For this purpose, the key ingredient is a transformation $\mathbf{S}(\underline{y}_k^1, \ldots, \underline{y}_k^m)$ of the measurements that is symmetric, i.e.,

$$\mathbf{S}\Big(\underline{y}_k^1, \ldots, \underline{y}_k^m\Big) = \mathbf{S}\Big(\underline{y}_k^{\pi_k(1)}, \ldots, \underline{y}_k^{\pi_k(m)}\Big) \tag{5}$$

for all feasible measurement-to-landmark associations π_k. A symmetric transformation allows to formulate the following new measurement equation

$$\underbrace{\mathbf{S}\Big(\underline{y}_k^1, \ldots, \underline{y}_k^m\Big)}_{=:\underline{z}_k} = \underbrace{\mathbf{S}\Big(h^1(\underline{x}_k, \underline{w}_k^1), \ldots, h^m(\underline{x}_k, \underline{w}_k^m)\Big)}_{=:g(\underline{x}_k, \underline{w}_k)} \ , \tag{6}$$

which does not depend on the unknown data association π_k, i.e., the ordering of the measurements in the pseudo-measurement $\mathbf{S}(\underline{y}_k^1, \ldots, \underline{y}_k^m)$ is irrelevant due to the symmetry of $\mathbf{S}$. Hence, there is no data association uncertainty anymore, but additional nonlinearity is introduced.

The above measurement equation (6) can be written in the compact form

$$\underline{z}_k = g(\underline{x}_k, \underline{w}_k) \ , \tag{7}$$

in which a reformulated measurement equation $g(\underline{x}_k, \underline{w}_k)$ maps the state and the noise term $w_k := \left[(\underline{w}_k^{L_1})^T, \ldots, (\underline{w}_k^{L_m})^T \right]^T$ to a pseudo-measurement $\underline{z}_k$.

It is important to note that the dimension of a pseudo-measurement should be at least $m \cdot d_y$ in order to ensure that all information from the original measurements are captured.

Based on (7), nonlinear Kalman filters such as the Extended Kalman Filter (EKF) or Unscented Kalman Filter (UKF) [19], [20] can be used for performing the measurement update.

It is clear that the specific form of the symmetric transformation is important. When using a nonlinear Kalman filter, the symmetric transformation should be as "linear" as possible. In the following, we discuss two specific types of symmetric transformations that have been used in the literature.

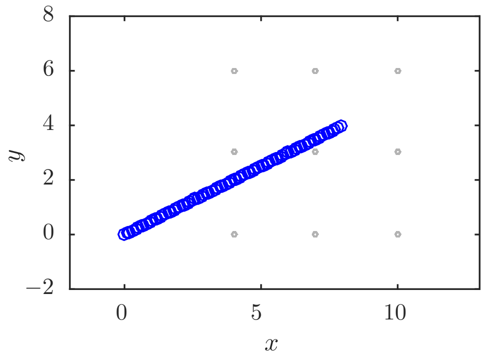


Fig. 1: Map with landmarks (grey dots) and robot trajectory (blue circles) for 80 time steps.

A. Symmetric Polynomial Transformations

The original SME approach employs polynomial symmetric functions. For example, the *Sum-Of-Powers* [14], [15], [19], [20] transformation for two landmarks and two one-dimensional measurements y_k^1 and y_k^2 becomes

$$\mathbf{S}\left(y_k^1, y_k^2\right) = \begin{pmatrix} y_k^1 + y_k^2 \\ (y_k^1)^2 + (y_k^2)^2 \end{pmatrix} \in \mathbb{R}^2 \ .$$

Unfortunately, polynomial symmetric functions become quite complex for higher dimensions and the degree of the polynomials increases with the number of landmarks.

B. Symmetric Kernel Transformation

As an alternative to polynomial symmetric transformations, we proffer the recently introduced concept of Kernel-SMEs [2], [17]. The key idea is to interpret the measurements as a Gaussian mixture function

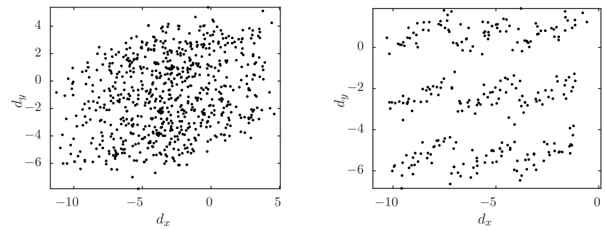
$$F_{\underline{z}}\left(\underline{y}_k^1, \ldots, \underline{y}_k^m\right) = \sum_{i=1}^{m} \mathcal{N}(\underline{z} - \underline{y}_k^i, \Gamma) \quad (8)$$

depending on the parameter $\underline{z}$, where $\mathcal{N}(\underline{z} - \underline{y}_k^i, \Gamma)$ is a Gaussian kernel with a suitable width Σ. This function is symmetric, one-dimensional, and depends on the free parameter $\underline{z}$.

We propose to evaluate (8) at specific instantiations of $\underline{z}$ in order to generate a pseudo-measurement with a sufficient high dimension. For this purpose, we choose n_a test vectors $\underline{a}^1, \ldots, \underline{a}^{n_a} \in \mathbb{R}^{d_y}$ in order construct the final measurement equation

$$\mathbf{S}\left(\underline{y}_k^1, \ldots, \underline{y}_k^m\right) = \begin{pmatrix} \mathbf{F}_{\underline{a}^1}\left(\underline{y}_k^1, \ldots, \underline{y}_k^m\right) \\ \vdots \\ \mathbf{F}_{\underline{a}^{n_a}}\left(\underline{y}_k^1, \ldots, \underline{y}_k^m\right) \end{pmatrix} \ . \quad (9)$$

The locations of the test vectors should represent the original Gaussian mixture function (8) as good as possible. In order to achieve this, for each measurement, we choose $2 \cdot d_y$ testpoints according to the sampling rule of the Unscented



(a) Scenario 1. (b) Scenario 2.

Fig. 2: Example measurements received from all time steps.

Kalman filter [11]. Hence, the test vectors $\underline{a}_k^1, \ldots, \underline{a}_k^{n_a}$ with $n_a = 2 \cdot d_y \cdot m$ are

$$\underline{a}_k^{l+i-1} := \underline{y}_k^l + (\sqrt{d_y \Gamma})_i \quad (10)$$

$$\underline{a}_k^{l+(i-1)+d_y} := \underline{y}_k^l - (\sqrt{d_y \Gamma})_i \quad (11)$$

for $i = 1, \ldots, d_y$ and $\left(\sqrt{d_y \Gamma}\right)_i$ denotes the i-th column of $\sqrt{d_y \Gamma}$.

In [2], [17], analytic expressions are derived for the Linear Minimum Mean Square Estimator (LMMSE) based on the kernel transformation (9) for general linear measurement and system equations. For nonlinear measurement or system equations, one can employ an approximate nonlinear Kalman filter such as the the Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF) [11], or S2KF [27]. Also, approximate nonlinear Kalman filter approaches might be computationally more efficient than the analytic expressions derived in [2], [17].

The kernel transformation comes with the advantage that it can easily be constructed for a large number of high-dimensional landmarks. The Gaussian kernel shape ensures that only the important regions of the measurement space are incorporated. On the downside, the kernel transformation requires to choose several parameters, i.e., the kernel width and the test points.

IV. Experiments

In this section, we investigate the benefits of the SME approach to Kalman-filter based SLAM. For this purpose, we consider a simple SLAM scenario in which the map consists of 9 two-dimensional landmarks.

In order to model the two-dimensional motion of the robot, we use a nearly constant velocity model, i.e., the robot state is four-dimensional and consists of the position $\left[x_k^R, y_k^R\right]^T$ and a velocity vector $\left[\dot{x}_k^R, \dot{y}_k^R\right]^T$ according to

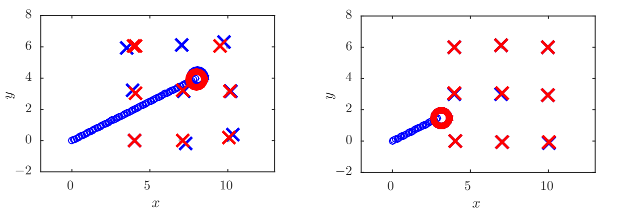
$$\underline{x}_k^R = \left[x_k^R, \dot{x}_k^R, y_k^R, \dot{y}_k^R\right]^T \in \mathbb{R}^4 \ . \quad (12)$$

The temporal motion is specified by the linear equation

$$\underline{x}_{k+1}^R = \mathbf{A}_k \cdot \underline{x}_k^R + \underline{v}_k^R \quad (13)$$

with system matrix

$$\mathbf{A}_k = \mathbf{I}_2 \otimes \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \ ,$$



(a) Scenario 1. (b) Scenario 2.

Fig. 3: Example run: Estimated landmark positions and robot location after 80 (Scenario 1) and 30 (Scenario 2) time steps. Results are shown for the Kernel-SME (blue) and NN (red).

and system noise covariance $\mathbf{C}^{v^R} = \mathbf{I}_2 \otimes \mathbf{Q}$ with $\mathbf{Q} = q_0 \begin{pmatrix} \frac{1}{3}T^3 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & T \end{pmatrix}$, where $\otimes$ denotes the Kronecker symbol, $\mathbf{I}_2$ is the two-dimensional identity matrix, $T = 1$ is the sampling period, and $q_0 = 0.000001$.

We assume that the robot observes the distance and angle to a landmark encoded in a two-dimensional Cartesian vector, i.e., the measurement equation (4) is

$$\underline{y}_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \underline{x}_k^R - \underline{x}_k^{L_i} + \underline{w}_k^{L_i} \tag{14}$$

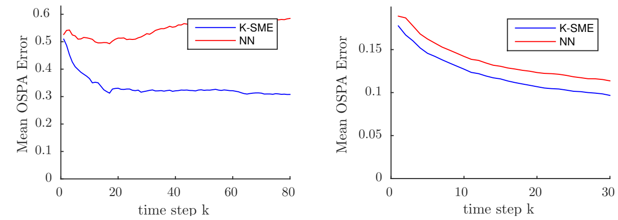
with additive zero-mean Gaussian noise $\underline{w}_k^{L_i}$ with covariance $\mathbf{D}_k^{L_i}$.

We employ the kernel transformation (9) with kernel width $\Gamma = 2$ and use the nonlinear Kalman filter *S2KF* [27] (with 50 samples per dimension) for inference. The SME-SLAM approach is compared with the Nearest Neighbor (NN) association, which finds the associates the measurement with the lowest cost (w.r.t. the Mahalanobis distance) to a landmark [25]. Fig. 1 depicts the landmarks and the robot trajectory.

The prior uncertainty of the robot state is given by $\mathbf{C}_0^R = \text{diag}([0.5, 0.5, 0.1, 0.1])$ and the prior uncertainty of the landmarks is denoted as $\mathbf{C}_0^{L_i}$, where we consider two different settings:

- *Setting 1 (80 time steps)*
  Prior landmark covariance $\mathbf{C}_0^{L_i} = 0.17\mathbf{I}_2$; large measurement noise $\mathbf{D}_k^{L_i} = 0.5\mathbf{I}_2$.
- *Setting 2 (30 time steps)*
  Prior landmark covariance $\mathbf{C}_0^{L_i} = 0.02\mathbf{I}_2$; medium measurement noise $\mathbf{D}_k^{L_i} = 0.1\mathbf{I}_2$.

An impression of the magnitude of the measurement noise is given in Fig. 2a and Fig. 2b, where measurements from example runs are depicted. The quality of an estimated map is assessed with the Optimal Sub-Pattern Assignment (OSPA) distance [26] averaged over 50 runs, see Fig. 4. For each run, the mean of the initial estimate is randomly drawn from a normal distribution with covariance $\mathbf{C}_0^R$ and $\mathbf{C}_0^{L_i}$. Examples of the final estimated maps are shown in Fig. 3. It can be seen that the Kernel-SME SLAM approach outperforms the NN approach in these settings. The reason is that the prior uncertainties of the landmarks are rather high so that the



(a) Scenario 1. (b) Scenario 2.

Fig. 4: Average error of the estimated landmark locations.

NN might choose the wrong associations in the beginning, which leads to inconsistent maps, see Fig. 3a. Nevertheless, we note that the NN will always outperform the SME-SLAM approach in case of low noise. The reason is that the NN approach then finds the correct solution, while the SME approach still works with a nonlinear measurement equation.

V. Conclusions and Future Work

Data association is a challenging problem in SLAM. For example, the number of possible association hypotheses grows exponentially with the number of landmarks, and in case of high measurement noise, there might be many feasible measurement-to-landmark associations.

In this work, we investigated a novel approach to deal with unknown data associations in Kalman filter-based SLAM. The key idea is to employ the Symmetric Measurement Equation (SME) approach, which was proposed in the context of multi-object tracking, in order to remove the data association uncertainty from the measurement equation. In combination with a sample-based nonlinear Kalman filter, this approach is extremely simple to implement and no (direct) additional computational complexity is introduced as one can immediately use the modified measurement equation. In general, the modified measurement equation is highly nonlinear; hence, sophisticated nonlinear estimators are required. Simulations show that the SME-SLAM method can outperform hard data association methods such as the simple Nearest Neighbor (NN) method. Future work will incorporate clutter measurements, i.e., false alarms, in the SME-SLAM approach.

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