Comparative Study of Track-to-Track Fusion Methods for Cooperative Tracking with Bearings-only Measurements

Susanne Radtke, **Kailai Li**, **Benjamin Noack**, and **Uwe D. Hanebeck**
Intelligent Sensor-Actuator-Systems Laboratory (ISAS)
Institute for Anthropomatics and Robotics
Karlsruhe Institute of Technology (KIT), Germany
susanne.radtke@kit.edu, kailai.li@kit.edu, benjamin.noack@ieee.org, uwe.hanebeck@ieee.org

***Abstract*—Using a network of spatially distributed sensors to track a moving object can be a challenging task. In applications with limited communication between sensor nodes and packet loss, it may be impossible to process measurements from these distributed sensor nodes in a central unit. Therefore, it is often necessary to use only the locally available measurements at the sensor nodes and afterwards merge all local tracks into one consistent result. In this paper, several different track-to-track fusion algorithms are compared to cooperatively track a moving object using only bearing measurements. It is shown that the Sample-based Fusion that uses a set of deterministic samples to reconstruct the cross-covariances is a suitable fusion algorithm for the considered setup. Furthermore, it provides the means to efficiently keep track of the cross-covariances between sensor nodes and therefore outperforms conservative methods. The proposed approach is also tested in a real-world indoor localization setup using bearings-only acoustic measurements from three microphone arrays.**

I. Introduction

Target tracking is an important task in the field of surveillance [1], where different sensors are used to obtain information about the current state of a moving target. Distance or angle measurements are often utilized for target tracking, which are related to nonlinear measurement equations. Possible setups could use passive sensors, such as acoustic sensors or electro-optical sensors that detect only the angle towards the target [2]. Bearing measurements are particularly challenging as they may not allow full observability of the target position [3]. Therefore, the Bearings-only Tracking (BoT) problem has been studied intensively during the last decades, resulting in a large number of different approaches and strategies, e.g., [4]–[7].

Using a centralized Kalman filter to obtain suitable results is not possible in all applications. Especially in scenarios with limited bandwidth and lossy communication, distributed estimation is a more robust, flexible, and modular solution [8]. In distributed estimation, a local Kalman filter is used to process the locally available sensor data. The quality of this locally estimated track can further be improved by fusing it with the tracks from other sensor nodes so that the fused result is more accurate and the uncertainty is reduced. This Track-to-Track Fusion (T2TF) problem is challenging since the local estimates are correlated due to common prior information, double counting, and the incorporation of common process noise [9]–[11]. By an adequate representation of these correlations, a bandwidth-efficient exchange of the required information among the nodes can be realized allowing various interesting applications.

In [12], a decentralized solution for tracking in a sensor network with varying coverage has been presented. The performance of various fusion methods has been compared, including Safe Fusion [13], Covariance Intersection (CI) [14], Inverse Covariance Intersection (ICI) [15], [16], and the Generalized Information Matrix Filter (GIMF) [17]. Many fusion methods, e.g., Ellipsoidal Intersection [18] and Covariance Intersection, employ an approximation or bound of the actual covariance matrix and may therefore be too optimistic or pessimistic, respectively. Keeping track of the cross-covariances yields optimal results but is cumbersome and often not feasible in sensor networks with many nodes and unreliable communication. Therefore, approaches to reconstruct the cross-covariances between state estimates in a distributed fashion were recently investigated to allow more accurate fusion results [19]. In [20], a set of deterministic samples was used to reconstruct the cross-covariances. This so-called Sample-based Fusion (SbF) method yields optimal results if all process noise terms are included and suboptimal results if the user-defined time horizon is chosen to be smaller [21].

This paper offers a comparative study for different T2TF methods used in an indoor tracking application, where a moving, sound-emitting object is cooperatively localized by a network of passive acoustic sensors in a distributed fashion. The problem of estimating these bearings-only measurements using passive acoustic sensors was introduced in [22]. The necessity for distributed estimation is induced by the limited feasibility of sending all acoustic measurements to a central processing unit. In order to process the acoustic measurements directly, the sensor nodes need to be synchronized accurately, which is not feasible in the considered setup. Moreover, the communication of the locally preprocessed bearing measurements is not appropriate since the loss of measurements can lead to significant performance drops and

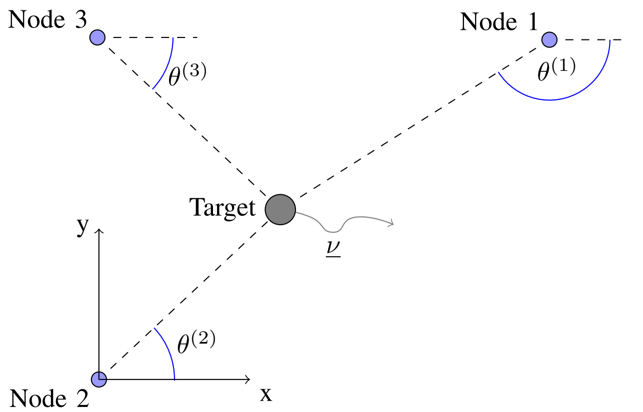


Fig. 1: Setup for the localization using three sensor nodes measuring the bearing $\theta \in [-\pi, \pi]$ towards a single target of velocity $\underline{\nu}$ in a joint global coordinate system.

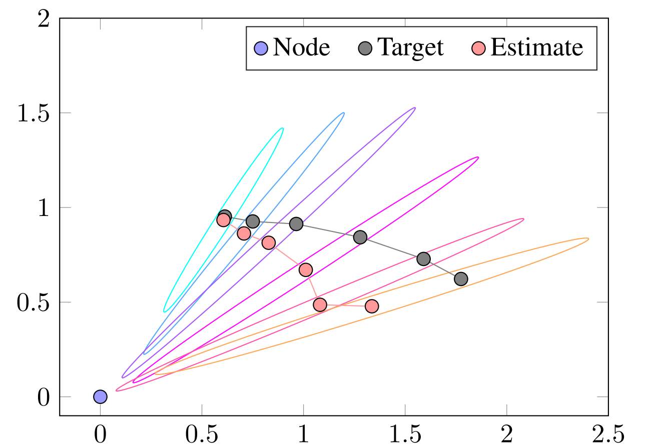


Fig. 2: Error ellipses and state estimate of a local Kalman filter over time when tracking a moving target going from left to right with bearings-only measurements at sensor node 2 of Fig. 1 at position $[0, 0]^{\mathrm{T}}$.

loss of tracks. Therefore, a distributed estimation approach with local estimators using the Unscented Kalman filter (UKF) [23] to handle the nonlinear measurements and a T2TF using the SbF is proposed.

The remainder of the paper is structured as follows. In Section II, the problem is formulated and different T2TF methods are briefly introduced. In Section III, the results of different fusion methods are evaluated based on both simulation and real data from an experimental setup. The results are concluded in Section IV and further research objectives are shown.

II. Problem Formulation

To increase robustness and flexibility for tracking in sensor networks, distributed estimation is used to obtain local tracks of the target using the locally available bearings-only measurements. These local tracks are later fused to obtain more accurate results. The problem of tracking a single target is divided into two parts. First, we will formulate a local tracker that will process only bearing measurements and prior information about the position of the target. Second, we will discuss the track-to-track fusion problem and give a short introduction to some state-of-the-art fusion methods.

A. Local Tracking

The target tracking algorithm will be developed for a sensor network with multiple sensor nodes measuring the bearing towards a moving object. For example, the nodes are equipped with microphone arrays measuring the direction, from which the target emits an acoustic wave front. Figure 1 shows the setup with three sensor nodes that measure the noisy angle $\theta^{(i)}$ towards the target.

The state of the moving object can be modeled as a discrete-time time-variant stochastic dynamic system

$$\underline{\boldsymbol{x}}_{k+1} = \mathbf{A}_k \underline{\boldsymbol{x}}_k + \mathbf{B}_k \underline{u}_k + \underline{\boldsymbol{w}}_k , \text{ with } \underline{\boldsymbol{w}}_k \sim \mathcal{N}(\underline{0}, \mathbf{Q}_k) ,$$

with state matrix $\mathbf{A}_k$ and input matrix $\mathbf{B}_k$, state vector $\underline{\boldsymbol{x}}_k$ with state dimension N, and input vector $\underline{u}_k$. The system is disturbed by white Gaussian system noise $\underline{\boldsymbol{w}}_k$ with covariance matrix $\mathbf{Q}_k$. The measurement model of sensor node i is given by the nonlinear function

$$\boldsymbol{y}_k^{(i)} = h^{(i)}(\underline{\boldsymbol{x}}_k) + \underline{\boldsymbol{v}}_k^{(i)} = z_k^{(i)} + \underline{\boldsymbol{v}}_k^{(i)} , \; \underline{\boldsymbol{v}}_k^{(i)} \sim \mathcal{N}(0, R_k^{(i)}) , \quad (1)$$

with additive white Gaussian noise $\underline{\boldsymbol{v}}_k$ with covariance matrix R_k. Since the sensors in this application measure the bearing towards the target, the measurement is the angle

$$z_k^{(i)} = \theta_k^{(i)} = \text{atan2}\left(\boldsymbol{x}_{y,k} - P_y^{(i)}, \boldsymbol{x}_{x,k} - P_x^{(i)}\right) ,$$

which is obtained from the target position $\underline{\boldsymbol{x}} = [\boldsymbol{x}_{x,k}, \boldsymbol{x}_{y,k}]^{\mathrm{T}}$ with respect to a local sensor node i at position $[P_x^{(i)}, P_y^{(i)}]^{\mathrm{T}}$. For each sensor node, the local angular measurements are all computed with respect to the same global coordinate system. Recursive estimation using bearings-only measurements is a highly nonlinear problem. In order to address these nonlinearities, various algorithms based on the Kalman filter have been developed. The most common approaches include the Extended Kalman Filter (EKF) [24] and the Unscented Kalman Filter (UKF) [23]. The UKF offers better accuracy in the considered bearings-only setup than the linearization of the EKF, because it accounts for the asymmetry of the nonlinear transformation [2]. Therefore, the UKF is used in this application to locally track the target. To compare the fused results in the evaluation part of this paper, a global unscented Kalman filter will be used additionally, which fuses the angle measurements from all sensor nodes. It should be noted, that due to the limited bandwidth and possible package loss this is not a suitable solution for the considered application. Figure 2 shows the result of the local unscented Kalman filter when tracking a moving target for several time steps from one sensor node. The estimation of the angle towards the target yields sufficiently accurate results, but the uncertainty for the distance grows very fast since no information about the distance can be obtained using only angle measurements. However, it can also be seen that the Gaussian uncertainty characterization does not capture the actual uncertainty and therefore, a systematic error between the target position and the estimate is introduced. The limitation of this local estimation can be overcome by using the information of other sensor nodes to achieve more accurate tracks.

B. Track-to-Track Fusion (T2TF)

Because of the one-dimensional bearings-only measurements, the estimates of a local tracker might diverge and get lost at some point. Since there are multiple local estimators that are tracking the target from different angles, it is possible to perform T2TF. The fused results can then be utilized to reinitialize the local estimators to enhance the performance of the filter and to prevent loss of track. There are various methods that are able to perform T2TF. In the well known Bar-Shalom/Campo formulas [25], the fusion rule to optimally merge two tracks into a consistent result were stated. These formulas were further extended for the multi-sensor case in [26]. The fusion result can be obtained by using the joint state estimate

$$\underline{\hat{m}}_{k|k} = \left[\left(\underline{\hat{x}}_{k|k}^{(1)}\right)^{\mathrm{T}} \quad , \ldots , \quad \left(\underline{\hat{x}}_{k|k}^{(L)}\right)^{\mathrm{T}} \right]^{\mathrm{T}}$$

and the joint cross-covariance

$$\mathbf{J}_{k|k} = \begin{bmatrix} \mathbf{P}_{k|k}^{(1)} & \mathbf{P}_{k|k}^{(1,2)} & \cdots & \mathbf{P}_{k|k}^{(1,L)} \\ \mathbf{P}_{k|k}^{(2,1)} & \mathbf{P}_{k|k}^{(2)} & \cdots & \mathbf{P}_{k|k}^{(2,L)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{k|k}^{(L,1)} & \mathbf{P}_{k|k}^{(L,2)} & \cdots & \mathbf{P}_{k|k}^{(L)} \end{bmatrix} ,$$

where

$$\mathbf{P}_{k|k}^{(i,j)} = E[(\underline{\hat{x}}_{k|k}^{(i)} - \underline{x}_k)(\underline{\hat{x}}_{k|k}^{(j)} - \underline{x}_k)^{\mathrm{T}}]$$

is a local covariance or cross-covariance matrix for $(i) = (i,i)$ or $(i,j), i \neq j$, respectively. Afterwards, the fused covariance

$$\mathbf{P}_{k|k} = \left(\mathbf{H}^{\mathrm{T}} \left(\mathbf{J}_{k|k}\right)^{-1} \mathbf{H} \right)^{-1} , \quad (2)$$

and the fused state

$$\underline{\hat{x}}_{k|k} = \mathbf{P}_{k|k} \mathbf{H}^{\mathrm{T}} \left(\mathbf{J}_{k|k}\right)^{-1} \underline{\hat{m}}_{k|k} \, . \quad (3)$$

can be calculated using the matrix $\mathbf{H} = [\mathbf{I}, \ldots, \mathbf{I}]^{\mathrm{T}}$ with the identity matrix $\mathbf{I}$ of the state dimension that describes how the local states map into the fused state estimate. Since the cross-covariances between the state estimates usually are unknown, there are several techniques to either bound or reconstruct the cross-covariances. The simplest approach is to simply neglect all correlations. This naïve fusion approach usually leads to very poor fusion results, because the uncertainty between the state estimates is underestimated. Therefore, other approaches such as Covariance Intersection, Inverse Covariance Intersection, and the Sample-based Fusion are considered.

1) Covariance Intersection (CI): Covariance Intersection is a very useful fusion method, since no knowledge about the underlying correlations between state estimates is required. The result is a convex combination of both state estimates of sensor node A and B that is composed by a scalar weighting factor $\omega \in [0,1]$. The fused state estimate and covariance matrix are calculated with

$$\mathbf{P}_{\mathrm{CI}} = \left(\omega (\mathbf{P}^A)^{-1} + (1-\omega)(\mathbf{P}^B)^{-1} \right)^{-1} ,$$

$$\underline{\hat{x}}_{\mathrm{CI}} = \mathbf{P}_{\mathrm{CI}} \left(\omega (\mathbf{P}^A)^{-1} \underline{\hat{x}}^A + (1-\omega)(\mathbf{P}^B)^{-1} \underline{\hat{x}}^B \right) .$$

Covariance Intersection has shown to be consistent under all possible correlations, but is also overly pessimistic in some applications. Therefore, other approaches try to utilize some additional information to obtain tighter bounds of the covariance matrix.

2) Inverse Covariance Intersection (ICI): Inverse Covariance Intersection [15], [16] yields a less conservative fusion result than Covariance Intersection by striving to find the maximum possible common information between the state estimates that are to be fused. To guarantee consistency, the possibly shared common information is bounded and removed from the fusion result. The fused covariance matrix can be calculated by

$$\mathbf{P}_{\mathrm{ICI}}^{-1} = (\mathbf{P}^A)^{-1} + \left(\mathbf{P}^B\right)^{-1} - \left(\omega \mathbf{P}^A + (1-\omega)\mathbf{P}^B\right)^{-1} .$$

Afterwards, the fused state estimated is calculated as a weighted combination of the local state estimates

$$\underline{\hat{x}}_{\mathrm{ICI}} = \mathbf{K}_{\mathrm{ICI}} \, \underline{\hat{x}}^A + \mathbf{L}_{\mathrm{ICI}} \, \underline{\hat{x}}^B ,$$

with weights $\mathbf{K}_{\mathrm{ICI}}$ and $\mathbf{L}_{\mathrm{ICI}}$ according to

$$\mathbf{K}_{\mathrm{ICI}} = \mathbf{P}_{\mathrm{ICI}} \left((\mathbf{P}^A)^{-1} - \omega \left(\omega \mathbf{P}^A + (1-\omega)\mathbf{P}^B \right)^{-1} \right) ,$$

$$\mathbf{L}_{\mathrm{ICI}} = \mathbf{P}_{\mathrm{ICI}} \left((\mathbf{P}^B)^{-1} - (1-\omega) \left(\omega \mathbf{P}^A + (1-\omega)\mathbf{P}^B \right)^{-1} \right) .$$

The properties of ICI are still an ongoing research objective with many promising applications for typical Kalman filter-based fusion problems.

3) Sample-based Fusion (SbF): The Sample-based Fusion is a relatively new method to perform T2TF utilizing a set of deterministic samples. It is able to reconstruct the cross-correlation between the tracks in a distributed fashion and therefore enables the use of the Bar-Shalom/Campo formulas. The usage of samples also allows a straightforward approach for nonlinear filters [20], [21]. In the beginning, an identity set is created using the simple deterministic spherical simplex sampling method described in [27], resulting in the sample set $\{\underline{p}^{(m)}\}_{m=1}^{M}$ with

$$\sum_{m=1}^{M} \underline{p}^{(m)} = \underline{0} \, , \; \sum_{m=1}^{M} \underline{p}^{(m)} \left(\underline{p}^{(m)}\right)^{\mathrm{T}} = \mathbf{I}_{D \times D} \, ,$$

with dimension $M = D + 1 = N \times (\mathcal{T} + 1) + 1$ where $\mathcal{T}$ is a user-defined time-horizon which denotes how many noise terms are included in the sample set. The Sample-based Fusion is basically a square root decomposition of the underlying covariance matrix. The included noise terms have to be factorized via Cholesky decomposition into the following form

$$\mathbf{\Sigma}_k = \mathrm{diag}\left(\sqrt{\mathbf{P}_{k|k}}, \sqrt{\mathbf{Q}_{k+1}}, \ldots, \sqrt{\mathbf{Q}_{k+\mathcal{T}}} \right) ,$$

such that the sample set can be initialized as

$$\begin{aligned}\underline{d}_k^{(m)} &= \mathbf{\Sigma}_k \underline{p}^{(m)} \ , \ \ \forall m = 1, \dots, M \\ &= \left[(\underline{s}_{k|k}^{(i,m)})^{\mathrm{T}}, \quad (\underline{w}_{k+1}^{(m)})^{\mathrm{T}}, \quad \dots, \quad (\underline{w}_{k+\mathcal{T}}^{(m)})^{\mathrm{T}} \right]^{\mathrm{T}} .\end{aligned}$$

This results in a weighted sample set $\{\underline{d}_k^{(m)}\}_{m=1}^M$, where $\{\underline{s}_{k|k}^{(i,m)}\}_{m=1}^M$ denotes the common prior information between sensor nodes and the $\{\underline{w}_{k+1,\dots,\mathcal{T}}^{(m)}\}_{m=1}^M$ denotes the uncorrelated process noise terms until the time horizon $\mathcal{T}$.

The cross-covariance information can be encoded into the sample set by performing the time update step using the system equation

$$\underline{s}_{k|k-1}^{(i,m)} = \mathbf{A}_k \underline{s}_{k-1|k-1}^{(i,m)} + \underline{w}_k^{(m)} \ , \ \ \forall m = 1, \dots, M \, ,$$

and afterwards the measurement update is performed. By using the nonlinear transform of the UKF, the sample set is propagated during the measurement update [20] by

$$\underline{s}_{k|k}^{(i,m)} = \left(\mathbf{I} - \mathbf{P}_k^{xy} (\mathbf{P}_k^{y})^{-1} (\mathbf{P}_k^{xy})^{\mathrm{T}} (\mathbf{P}_k^{x})^{-1} \right) \underline{s}_{k|k-1}^{(i,m)} \, ,$$

where $\mathbf{P}_k^y$ is the covariance of the predicted measurement, $\mathbf{P}_k^{xy}$ is the cross-covariance between the predicted state and the predicted measurement, and $\mathbf{P}_k^x = \mathbf{P}_{k|k-1}$ is the covariance of the predicted state estimate. To perform the fusion step, the cross-covariance terms at time horizon $\mathcal{T}$ are calculated using

$$\mathbf{P}_{k+\mathcal{T}|k+\mathcal{T}}^{i,j} = \sum_{m=1}^{M} \underline{s}_{k+\mathcal{T}|k+\mathcal{T}}^{(i,m)} \big(\underline{s}_{k+\mathcal{T}|k+\mathcal{T}}^{(j,m)} \big)^{\mathrm{T}} .$$

The obtained cross-covariance matrices are then used in the fusion equations (2) and (3), which lead to the fused state estimate and covariance matrix.

III. Evaluation

In this section, the proposed distributed estimation and T2TF are evaluated. First, the results are simulated to allow an empirical comparison of the used fusion methods. Second, real data from a moving, sound-emitting target that is tracked by local microphones is evaluated.

A. Simulation results

The motion of the target is modeled with a time-invariant stochastic time-discrete system equation

$$\underline{x}_{k+1} = \mathbf{A} \underline{x}_k + \underline{w}_k, \ \text{with} \ \underline{w}_k \sim \mathcal{N}(\underline{0}, \mathbf{Q}) \, ,$$

with additive white Gaussian process noise $\underline{w}_k$ and covariance matrix $\mathbf{Q}$. We are using a constant velocity model

$$\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}_{k} + \underline{w}_k \, ,$$

with $\Delta T = 0.1$ and additive white Gaussian process noise $\underline{w}_k$ with covariance matrix

$$\mathbf{Q} = q \begin{bmatrix} \frac{1}{3}\Delta T^3 & 0 & \frac{1}{2}\Delta T^2 & 0 \\ 0 & \frac{1}{3}\Delta T^3 & 0 & \frac{1}{2}\Delta T^2 \\ \frac{1}{2}\Delta T^2 & 0 & \Delta T & 0 \\ 0 & \frac{1}{2}\Delta T^2 & 0 & \Delta T \end{bmatrix} .$$

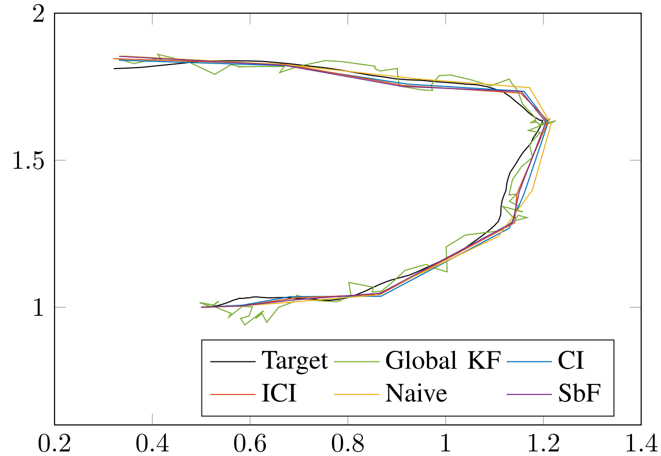


Fig. 3: Comparison of the simulated track with the estimated track of the global Kalman filter and the fused tracks of different fusion methods.

The noise power is assumed to be $q = 0.01$ and the covariance $R = \sigma^2 = (2 \cdot \frac{\pi}{180})^2$ of the measurement noise $\underline{v}_k^{(i)}$ in (1) has been determined from the experiment with real sensor data obtained by the microphone array. The target was observed from three sensor nodes with node 1 at $[2.98, 3]^{\mathrm{T}}$, node 2 at $[0, 0]^{\mathrm{T}}$ and node 3 at $[-0.01, 3.02]^{\mathrm{T}}$. The filters were initialized with state estimate $\hat{\underline{x}} = [1.5, 1.5, 0, 0]^{\mathrm{T}}$ and covariance matrix $\mathbf{P} = \mathbf{Q}$. The fusion step was performed every 10 time steps. The performance of the Sample-based Fusion (SbF), Covariance Intersection (CI), Inverse Covariance Intersection (ICI), and the naïve fusion, where all cross-correlations are ignored, is compared. Figure 3 shows an example of a simulated trajectory with the ground truth of the target and the results of the fusion methods. Additionally, the results of the global filter that utilizes all bearing measurements are shown. All methods can follow the target trajectory well. The track of the global filter seems noisier as it contains an estimation at every time step while the fused tracks only contain an estimation every fusion step. Figure 4 compares the Mean Squared Error (MSE) of the state estimates. It can be seen that the state estimates of all filters except the global Kalman filter diverge quickly until they are reinitialized with the fused state estimate after the fusion step. The MSE of the fused state estimates is shown in Figure 5(a). The evaluation example shows that the Sample-based Fusion results in the smallest MSE, followed by the Inverse Covariance Intersection, then the Covariance Intersection, and lastly the naïve fusion. To compare the fusion results in terms of consistency, the Average Normalized Estimation Error Squared (ANEES) is used [28], where N is the dimension of the system and n_{MCR} is the number of Monte Carlo runs

$$\bar{\epsilon} = \frac{1}{N n_{\text{MCR}}} \sum_{i=1}^{n_{\text{MCR}}} \epsilon_i = \frac{1}{N n_{\text{MCR}}} \sum_{i=1}^{n_{\text{MCR}}} (\hat{\underline{x}}_i - \underline{x}_i)^{\mathrm{T}} \mathbf{P}_i^{-1} (\hat{\underline{x}}_i - \underline{x}_i) \, .$$

The ANEES measures the credibility of an estimator that should be approximately 1, meaning that the estimated co-

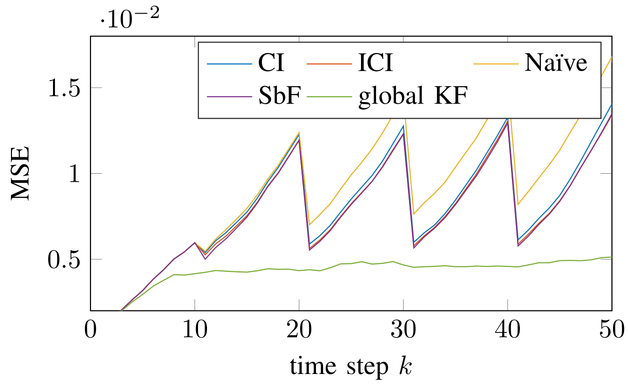
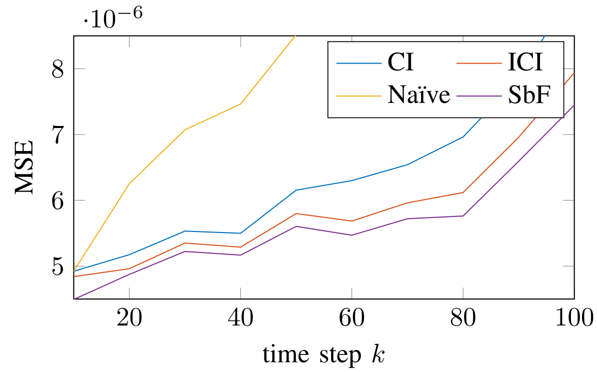


Fig. 4: Comparison of the Mean Squared Error (MSE) of the global Kalman filter and the local estimates of sensor node 1 using Covariance Intersection (CI), Inverse Covariance Intersection (ICI), naïve fusion (Naive) and the Sample-based Fusion (SbF).

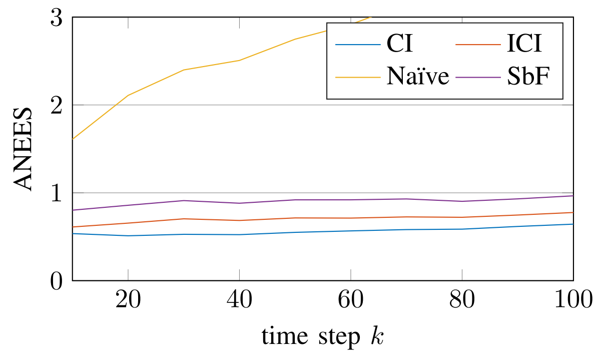
variance matrix matches the actual error. If it is higher than 1, then the uncertainty is underestimated. Conservative methods such as CI tend to overestimate the uncertainty, therefore achieving an ANEES smaller than 1. Figure 5(b) shows that the Sample-based Fusion results in an ANEES slightly smaller than 1, meaning that the error is not overly pessimistic, yet the result is consistent. The results of the naïve fusion show that ignoring the correlation between sensor nodes yields a too optimistic assessment of the estimation error. Both ICI and CI are conservative methods with CI being the most conservative, as expected.

B. *Experimental results with real data*

For the experimental results, a noise emitting object was tracked by three sensor nodes with node 1 at $[2.98\,\mathrm{m}, 3\,\mathrm{m}]^\mathrm{T}$, node 2 at $[0\,\mathrm{m}, 0\,\mathrm{m}]^\mathrm{T}$ and node 3 at $[-0.01\,\mathrm{m}, 3.02\,\mathrm{m}]^\mathrm{T}$, which corresponds to the same configuration as in the simulation. Each sensor node was equipped with a microphone array (see Figure 6) to determine the direction from which an acoustic wave front is approaching. For the emitted sound, electronic music with a constantly beating drum was chosen. The object was moving in a straight line. The initial position of the local state estimates was set to $[1.5\,\mathrm{m}, 2\,\mathrm{m}]^\mathrm{T}$ which is distant from the real position of the target to see how robust the proposed method works. The local estimates are updated every $0.1\,\mathrm{s}$ and the fusion is executed after $1\,\mathrm{s}$. Afterwards, the fusion results were used to reinitialize the local estimators. The results have been calculated offline. Therefore, it is also possible to use a global Kalman filter that utilizes all angle measurements and obtains a much more precise estimate. Figure 7 shows that the results of all methods converge rapidly towards the real trajectory of the object. Again, the results of the global Kalman filter appear noisier as the T2TF methods only acquire a result after every fusion step and therefore appear smoother. All methods except CI are robust against the wrong initial estimate.



(a) Mean Squared Error (MSE) from 1000 test runs.



(b) Average Normalized Estimation Error Squared (ANEES) from 1000 test runs.

Fig. 5: Comparison of the fused estimates of Covariance Intersection (CI), Inverse Covariance Intersection (ICI), naïve fusion (Naive) and the Sample-based Fusion (SbF).

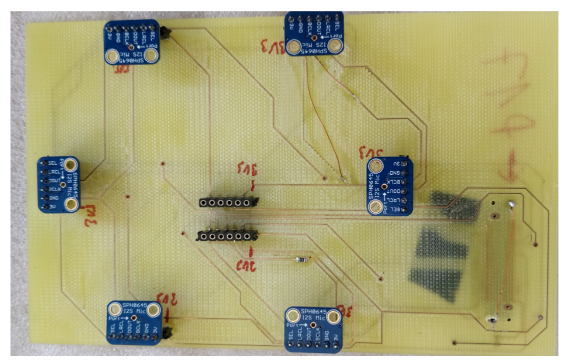


Fig. 6: Microphone array for measuring the angular locations of sound sources.

IV. Conclusion

This paper compared the performance of different T2TF methods used in a cooperative tracking application with only locally available bearing measurements. The results show that it is possible to use the proposed approach to track a moving target in a distributed fashion with only locally available bearings-only measurements. The Sample-based Fusion achieved the best results compared to the other tested T2TF methods. To use the method, a set of deterministic samples has to be communicated additionally to the state estimate

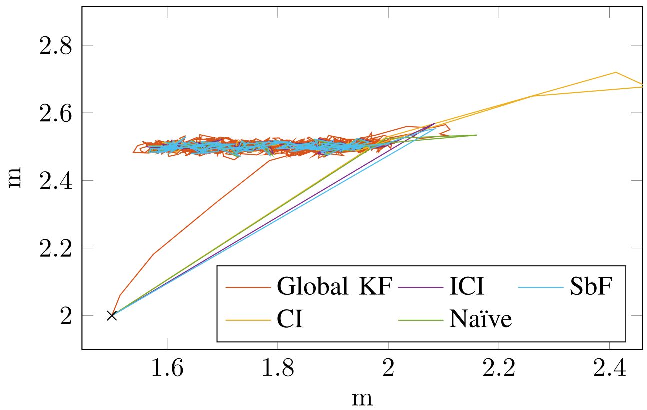


Fig. 7: Estimated track of a sound emitting target moving very slowly from right to left with wrong initial position (marked with x) to test robustness of the approach.

and the covariance matrix. Depending on the state space and the time horizon until the fusion takes place, this requires additional communication resources. Assumptions about the motion model of the target are an important aspect for the performance of the T2TF and the proposed solution is sensitive to maneuvering targets and model mismatch. The experiments with the microphone arrays also showed problems with clutter that could be addressed with data association. The periodic domain of the angular measurements could also be handled using directional estimation approaches [22]. This would lead to interesting new challenges for reconstructing the cross-covariances between state estimates. Further research may investigate the problem of bearings-only tracking with multiple targets in a distributed fashion.

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