Extended Object and Group Tracking: A Comparison of Random Matrices and Random Hypersurface Models

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Abstract: Based on previous work of the authors, this paper provides a comparison of two different tracking methodologies for extended objects and group targets, where the true shape of the extent is approximated by an ellipsoid. Although both methods exploit usual sensor data, i.e., position measurements of varying scattering centers, the distinctions are a consequence of the different modeling of the extent as a symmetric positive definite random matrix on the one hand and an elliptic random hypersurface model on the other. Besides analyzing the fundamental assumptions and a comparison of the properties of these tracking methods, simulation results are presented based on a static tracking environment to highlight especially the differences in the update step for the extension estimate.

1 Introduction

The explicit consideration of target extents is becoming more and more important in the development of modern tracking systems, where an object is considered as extended if it is the source of multiple measurements at the same time. The specific set of problems arises from different scattering centers, named measurement sources, which give rise to several distinct radar detections varying, from scan to scan, in both number as well as relative origin location. In order to improve robustness and precision of estimation results, it is felt desirable to track the target extent in addition to the kinematic state of the object. More than these quantities cannot safely be estimated as well in the (opposite) case, where limited sensor resolution causes a fluctuating number of detections for a group of closely spaced targets and thus, prevents a successful tracking of (all of) the individual targets. In this case, it is suitable to consider this group of point targets as a *single* extended object.

Several suggestions for dealing with this problem can be found in literature. For an early work, see [DBP90]. An overview of existing work up to 2004 is given in [WD04]. In general, implicit and explicit models for the scattering centers, i.e., the measurement sources, can be distinguished. Explicit target models [VIG04, VIG05, IG03] aim at estimating the location of the measurement sources in addition to kinematics of the object. These approaches require data association and are not treated in this paper. An implicit model for the target extent assumes that the measurement sources are generated independently according to an internal state of the object that reflects the extent. An example for an implicit model is a so-called spatial distribution model, where each measurement source is assumed to be an independent random draw from a probability distribution [GGMS05, GS05].

In this paper, we discuss two different tracking methods for extended object tracking, where the measurement sources are modeled implicitly and the true physical extension is approximated by an ellipsoid. Ellipsoidal shapes are highly relevant for real world applications, as an ellipsoid provides useful information about the target orientation and spatial extent. Here, the focus is on track maintenance, while estimation under uncertain observation-to-track association in the possible presence of missed detections and false alarms is considered out of scope.

The first tracking method considered in this paper is based on the use of symmetric positive definite (SPD) random matrices and has been introduced in [Koc08]. Therein, an SPD random matrix to describe the ellipsoid is the counterpart of a random vector representing the centroid. The decisive point for tracking of extended objects in [Koc08] is the special interpretation of usual sensor data, whereupon each measurement is considered as a measurement of the centroid scattered over object extension. This implicit neglect of any statistical sensor error has caused some further investigations and developments to honor the fact that both sensor error and extension contribute to the measurement spread [FF09]. Hence, we employ two approaches: the original one of [Koc08] as well as the advanced one of [FF09]. The second tracking method is based on a random hypersurface model (RHM), which has been introduced in [BNH10]. An *Elliptic RHM* specifies the relative Mahalanobis distance of a measurement source to the center of the target object by means of a one-dimensional random scaling factor. In order to avoid the treatment of an SPD matrix for the ellipsoid, the kinematic state of the extended object is supplemented with the non-zero entries of the Cholesky factorization of the inverse SPD matrix. By this means, all parameters describing the corresponding ellipsoid are contained in one random vector.

This paper analyzes the fundamental assumptions of these three tracking approaches and provides a comparison of the properties of the two different methods. It is completed by simulation results based on a static tracking environment to highlight especially the differences in the update step for the extension estimate.

2 General Problem Description

The considered problem is to track the kinematics and the extent of an unknown extended object in the (x, y)-plane based on Cartesian position measurements corrupted by additive stochastic noise. It is assumed that there is a random number of n_k noisy position measurements arising from n_k unknown measurement sources in each scan k, i.e., the object extension is not directly observable. More formally, the unknown measurement source at time step t_k is denoted with $\mathbf{z}_k^j$. In this paper, the measurement $\mathbf{y}_k^j$ is the observation of $\mathbf{z}_k^j$

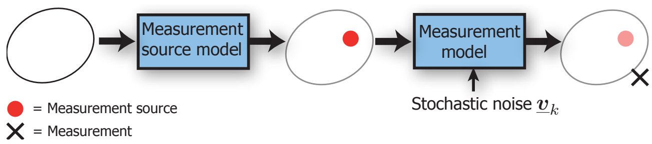

Figure 1: Model of the generation process for one measurement.

according to a specific measurement model (see Figure 1) given by

$$\mathbf{y}_k^j = \mathbf{z}_k^j + \mathbf{w}_k^j\,, \tag{1}$$

where $\mathbf{w}_k^j$ denotes additive white Gaussian noise. The goal is now to estimate and track a summarizing shape, e.g., an ellipsoid, which reflects the extent and shape of the measurement sources. In doing so, the major challenge is that the measurement sources are generated according to an unknown model. For instance, a group of point targets may generate at each time step only a discrete set of measurement sources. A spatially extended object, however, may generate measurement sources that stem from a continuous domain, i.e., the target surface. For ellipsoidal shapes, two different methods have been recently proposed: SPD random matrices (see Section 3) and *Elliptic RHMs* (see Section 4).

3 Method I: Random Matrices

In order to achieve robust tracking of extended objects and group targets, the two approaches in [Koc08, FF09] supplement the corresponding conventional (kinematic) track file with a random variable representing the physical extent, which is approximated by an ellipsoid with the result that the target extent can be described by a symmetric positive definite (SPD) matrix. Such a matrix-valued random variable is named a random matrix and invokes matrix-variate distributions [GN99].

3.1 Measurement Likelihood Function

According to the proposal in [Koc08, FF09], the physical extent represented by an SPD random matrix $\mathbf{X}_k$ is considered besides the kinematic state of the centroid described by the random vector $\mathbf{x}_k^T = [\mathbf{r}_k^T, \dot{\mathbf{r}}_k^T]$ with the spatial state component $\mathbf{r}_k$, where $d = \dim(\mathbf{r}_k)$ is also the dimension of $\mathbf{X}_k$. Now, it is assumed that in each scan k there is a random number of n_k independent position measurements

$$\mathbf{y}_k^j = \mathbf{H}\mathbf{x}_k + \mathbf{w}_k^j \tag{2}$$

with the measurement source $\mathbf{z}_k^j \mathrel{\widehat{=}} \mathbf{H}\mathbf{x}_k$ and the measurement matrix $\mathbf{H} = [\mathbf{I}_d, \mathbf{0}_d]$. Furthermore, expected sensor reports are considered as measurements of the centroid scattered over the extent so that, having regard to the measurement noise variance $\mathbf{R}$, the noise $\mathbf{w}_k^j$

is assumed to be a zero-mean normally distributed random vector with variance $z\mathbf{X}_k + \mathbf{R}$. The scaling factor z enables us to account for a difference between an assumed normal spread contribution of the extent and a possibly more realistic assumption such as, e.g., a uniform distribution of varying scattering centers over an object of finite extent. With this, the likelihood to measure the set $\mathbf{Y}_k := \{\mathbf{y}_k^j\}_{j=1}^{n_k}$ (denoting the set of the n_k measurements in a particular scan k) given both kinematic state and object extent as well as the number of measurements reads $p(\mathbf{Y}_k|n_k, \mathbf{x}_k, \mathbf{X}_k) = \prod_{j=1}^{n_k} \mathcal{N}(\mathbf{y}_k^j; \mathbf{H}\mathbf{x}_k, z\mathbf{X}_k + \mathbf{R})$. Thereupon, the computation of the mean measurement and the measurement spread

$$\overline{\mathbf{y}}_k = \frac{1}{n_k}\sum_{j=1}^{n_k} \mathbf{y}_k^j\,, \qquad \overline{\mathbf{Y}}_k = \sum_{j=1}^{n_k} (\mathbf{y}_k^j - \overline{\mathbf{y}}_k)(\mathbf{y}_k^j - \overline{\mathbf{y}}_k)^T \tag{3}$$

is considered as a preprocessing step to rewrite the likelihood function according to

$$p(\mathbf{Y}_k|n_k, \mathbf{x}_k, \mathbf{X}_k) \propto \mathcal{N}(\overline{\mathbf{y}}_k; \mathbf{H}\mathbf{x}_k, (z\mathbf{X}_k + \mathbf{R})/n_k)\, \mathcal{W}(\overline{\mathbf{Y}}_k; n_k - 1, z\mathbf{X}_k + \mathbf{R})\,, \tag{4}$$

where $\mathcal{W}(\mathbf{X}; m, \mathbf{C})$ denotes the *Wishart density* [GN99] of an SPD random matrix $\mathbf{X}$ with expected SPD matrix $m\mathbf{C}$.

3.2 Bayesian Inference

In this context, a tracking algorithm is an iterative updating scheme for conditional probability densities $p(\mathbf{x}_k, \mathbf{X}_k|\boldsymbol{\mathcal{Y}}_k)$ of the joint object state $(\mathbf{x}_k, \mathbf{X}_k)$ at each time t_k given the accumulated sensor data $\boldsymbol{\mathcal{Y}}_k := \{\mathbf{Y}_\varkappa, n_\varkappa\}_{\varkappa=0}^k$. For this reason, the derivation of explicit filter equations is based on the application of the concept of *conjugate priors*—the very same concept that constitutes one possible way of deriving the well-known Kalman filter equations for point source tracking—to the measurement likelihood function for receiving update equations and then complemented with an evaluation of the *Chapman-Kolmogorov theorem* for obtaining a recursive Bayesian estimation cycle.

It appears that for the likelihood $p(\mathbf{Y}_k|n_k, \mathbf{x}_k, \mathbf{X}_k)$, no conjugate prior can be found that is both independent of $\mathbf{R}$ and analytically traceable. For this reason, any statistical sensor error is ignored in [Koc08], i.e., setting $\mathbf{R} = \mathbf{0}$, to derive a closed-form solution within a Bayesian framework on the supposition that the spread of the measurements is dominated by the extent. This means, by implication, that the estimator of [Koc08] effectively estimates extent *plus* sensor error, while an equal Kalman gain (for the kinematics estimate) is effective in each spatial dimension because the filter cannot judge the *quality* of the measurements.

In view of these observations, the alternative approach in [FF09] adapts the original approach in [Koc08] using some careful approximations to honor the fact that both sensor error and extent contribute to the measurement spread. In detail, the updated estimate of the centroid kinematics is determined by using standard Kalman filter equations with the mean measurement $\overline{\mathbf{y}}_k$ and an approximation of the true innovation covariance. However, this way of proceeding requires knowledge of the true extent $\mathbf{X}_k$ so that it is assumed that

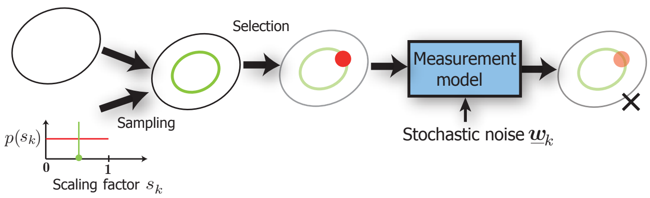

Figure 2: Random Hypersurface Models.

the predicted extent is not too far away from the truth and can be used as a replacement for $\mathbf{X}_k$ where needed.

In both approaches, the update of the extent estimate is carried out by a weighted sum of the predicted extent, the measurement spread, and a dyadic product, which evaluates the difference between the expected and the measured centroid position. The latter enables an update of the extent estimate even in the case of a single measurement. The differences between both approaches are due to the particular weighting of the individual quantities, where [FF09] uses matrix-valued scaling to compensate the effects of the statistical sensor error on the measurements.

4 Method II: Elliptic Random Hypersurface Models (RHMs)

The development of *Elliptic RHMs* for extended object tracking [BNH10] has been driven by the idea to estimate the smallest enclosing ellipse of the extended target.

An *Elliptic RHM* [BH09] is a specific measurement source model (see Figure 1) that assumes each measurement source to lie on a scaled version of the true ellipse describing the target (see Figure 2). The scaling factor is specified by an independent random draw from a one-dimensional probability density function. It can be interpreted as the (relative) distance of the measurement source from the target center with respect to the Mahalanobis distance induced by the true ellipse. It is important to note that all five ellipse parameters can be estimated based on measurements generated from an *Elliptic RHM*.

The probability density of the scaling factor has to be specified in advance. It was proven in [BNH10] that, if the measurement sources are drawn from a uniform spatial distribution on the entire ellipse surface, the *squared* scaling factor is uniformly distributed on the interval $[0, 1]$. Hence, a uniformly distributed *squared* scaling factor is a rational choice. Note that the converse is not true, i.e., a uniformly distributed squared scaling factor does *not* necessarily lead to a uniform distribution on the entire ellipse. There are many spatial distributions, which yield a uniformly distributed squared scaling factor.

Another natural choice is to assume the random scaling factor to be Gaussian distributed [BH09]. A Gaussian distributed scaling factor is for instance useful when measurement sources at the border of the ellipse are more probable than measurement sources in the center of the ellipse.

In order employ an *Elliptic RHM*, a suitable parametric representation of an ellipse has to be selected. The center of the ellipse is modeled with the random vector $\mathbf{r}_k$. In order to avoid the treatment of positive semi-definite random matrices, the shape of the ellipse, i.e., the physical extension of the extended object, is represented with the Cholesky decomposition of the SPD matrix $(\mathbf{X}_k)^{-1} = \mathbf{L}_k \cdot \mathbf{L}_k^T$, where

$$\mathbf{L}_k := \begin{bmatrix} \mathbf{a}_k & 0 \\ \mathbf{c}_k & \mathbf{b}_k \end{bmatrix} \tag{5}$$

is a lower triangular matrix with positive diagonal entries.

The state vector turns out to be of the form $\mathbf{x}_k = [\mathbf{r}_k^T, \mathbf{l}_k^T, \ldots]^T$ with $\mathbf{l}_k = [\mathbf{a}_k, \mathbf{b}_k, \mathbf{c}_k]^T$. Note that $\mathbf{x}_k$ may also contain further state variables, e.g., for the kinematics of the target. The ellipse specified by $\mathbf{x}_k^T$ is given by the set

$$\mathcal{E}_k = \{z \mid z \in \mathbb{R}^2 \text{ and } g(z, \mathbf{x}_k) = 1\}$$

with shape function $g(z, \mathbf{x}_k) := (z - \mathbf{r}_k)^T \cdot (\mathbf{L}_k \cdot \mathbf{L}_k^T) \cdot (z - \mathbf{r}_k)$. The scaled version of $\mathcal{E}_k$ with scaling factor $\mathbf{s}_k$ is given by

$$\mathcal{E}_k^s = \{z \mid z \in \mathbb{R}^2 \text{ and } g(z, \mathbf{x}_k) = \mathbf{s}_k^2\} \ .$$

4.1 Measurement Likelihood Function

The measurement likelihood function can be constructed by formulating a measurement equation, which relates the unknown state $\mathbf{x}_k$ to the measurement $\mathbf{y}_k^j$. If there would be no measurement noise, i.e., $\mathbf{y}_k^j = \mathbf{z}_k^j$, the measurement equation would be

$$g(\mathbf{z}_k^j, \mathbf{x}_k) - \mathbf{s}_k^2 = 0 \ , \tag{6}$$

which maps the unknown parameters $\mathbf{x}_k$ to the pseudo-measurement 0 with additive noise term $\mathbf{s}_k^2$. Unfortunately, the measurement source is not known, only its noisy measurement $\mathbf{y}_k^j = \mathbf{z}_k^j + \mathbf{w}_k^j$ is given. If the measurement $\mathbf{y}_k^j$ is inserted in (6), a deviation $\bar{\mathbf{w}}_k^j$ on the right hand side may be obtained, i.e., we obtain the measurement equation

$$g(\mathbf{y}_k^j, \mathbf{x}_k) - \mathbf{s}_k^2 = \bar{\mathbf{w}}_k^j \ . \tag{7}$$

Actually, $\bar{\mathbf{w}}_k^j$ is a random variable, because $\mathbf{y}_k^j$ depends on the measurement noise. The probability distribution of $\bar{\mathbf{w}}_k^j$ can be approximated with a Gaussian distribution by means of moment matching. For this approximation, the true parameters of the ellipse must be known. Hence, similar to the alternative random matrix approach [FF09], the true parameters are substituted with its current estimate. For the detailed formulas see [BNH10].

4.2 Bayesian Inference

A measurement update with the measurement equation (7) can in general be performed with a nonlinear state estimator. For both uniformly und Gaussian distributed scaling factors $\mathbf{s}_k^2$, closed-form expressions for the first two moments of the updated estimate can be derived (see [BNH10]). For this purpose, the probability distribution of the random vector $[\mathbf{x}_k^T, g(\mathbf{y}_k^j, \mathbf{x}_k) - \bar{\mathbf{w}}_k^j]^T$ is approximated with a Gaussian distribution by analytic moment matching. In case of a Gaussian distributed squared scaling factor $\mathbf{s}_k^2$, the updated estimate then results from the Kalman filter equation. For a uniformly distributed squared scaling factor $\mathbf{s}_k^2$, the updated estimate results from calculating the first two moment of a truncated Gaussian distribution.

Again, the measurement update step is complemented with an evaluation of the *Chapman-Kolmogorov theorem* for obtaining a recursive Bayesian estimation cycle.

5 Comparison

This section highlights the differences of both tracking methods for tracking extended objects. For this purpose, we directly compare several properties that characterize both methods. Subsequently, the main differences of both methods are discussed.

5.1 Method I: Random Matrices

Representation of an ellipse. The basic assumption to model the extent by an ellipsoid corresponds with earlier work (see, e.g., [Bla86, DBP90]), but the novelty of [Koc08, FF09] is that it relies on the joint estimation of centroid kinematics and physical extent. For that purpose, an SPD random matrix to describe this ellipsoid is the counterpart of a random vector representing the centroid and enables us to derive closed-form expressions for the Bayesian inference mechanism.

Interpretation of an elliptic extent. For a group of closely spaced targets, the obtained tracking results are comparable to the sample mean and the (scaled) sample covariance of the group members.

Modeled measurement source distributions. By means of the scaling factor z, it is possible to account for a difference between an assumed normal spread contribution of the extent and a possibly more realistic assumption like, e.g., a uniform distribution of varying scattering centers over an object of finite extent $\mathbf{X}$. In this instance, recalling the idea of second order moment matching leads to $z = 1/4$ as an appropriate choice for the scaling factor because the variance of the varying measurement sources is equal to $\mathbf{X}/4$.

Processing of measurements. Both random matrix approaches update the state estimates by exploiting indirect measurement parameters: the mean measurement $\overline{\mathbf{y}}_k$ and the measurement spread $\overline{\mathbf{Y}}_k$. This means that all n_k measurements are processed in one update step, where the computation of $\overline{\mathbf{y}}_k$ and $\overline{\mathbf{Y}}_k$ can be considered as a preprocessing step.

Other target shapes. The use of SPD random matrices confines the target shape modeling to ellipsoids, where an eigenvalue corresponds to the squared semi-axis length.

Estimation of measurement noise. In the case of point source targets, it is possible for both approaches to estimate an unknown measurement error covariance (i.e., setting $\mathbf{R} = \mathbf{0}$ in the alternative approach [FF09]) by means of the dyadic product, which evaluates the difference between the expected and the measured centroid position.

5.2 Method II: Elliptic RHM

Representation of an ellipse. An *Elliptic RHM* represents an ellipse by means of a parameter vector consisting of the center and the vectorized Cholesky decomposition of the SPD shape matrix. The uncertainty about the parameter vector of the ellipse is modeled with a Gaussian distribution. However, RHMs are not restricted to this particular representation of the parameters.

Interpretation of an elliptic extent. An *Elliptic RHM* aims at estimating the smallest enclosing ellipse of the extended target. One can say that the parameters of the ellipse are estimated such that the squared relative Mahalanobis distance from the center to the measurement sources is uniformly distributed. For a group of closely spaced targets, the center of this ellipse does not have to be the sample mean and the shape matrix does not have to coincide with the (scaled) sample covariance of the group members.

Modeled measurement source distributions. An *Elliptic RHM* with uniformly distributed squared scaling factor is able to estimate the correct parameters of an ellipse on which the measurement sources are drawn from a uniform distribution. However, an *Elliptic RHM* with uniformly distributed squared scaling factor models in fact many spatial distributions on the ellipse.

Processing of measurements. The measurement update step for *Elliptic RHM* does not make use of a preprocessing step. Independent measurements can be processed sequentially. Moreover, the computational complexity is linear in the number of received measurements.

Other target shapes. RHMs are modular. The parametric representation of the shape and the inference mechanism can be exchanged. RHMs can even be generalized to other shapes by means of employing a proper implicit shape function.

Estimation of measurement noise. In general, an *Elliptic RHM* could also be used for estimating the measurement noise of a point target. An *Elliptic RHM* can also be seen as a special type of noise, which specifies the random distance of a disturbance. In order to estimate Gaussian noise, the corresponding probability distribution of the scaling factor would have to be employed. However, this has not been investigated so far.

5.3 Discussion

The first main difference between the two methods is the interpretation of the ellipsoidal extent. While for a group of point source targets, the tracking results of the random matrix approaches are comparable to the sample mean vector and the (scaled) sample covariance matrix of the group members, *Elliptic RHMs* aim at estimating the smallest enclosing ellipse of the target group. However, if the measurement sources are drawn uniformly over an ellipse, both tracking methods estimate the same ellipse. The random matrix approaches are also capable of estimating Gaussian distributed measurement sources. On the other hand, *Elliptic RHMs* are able to employ Gaussian distributed scaling factors. The second main difference is the representation of an ellipse. The random matrix method employs an SPD random matrix for the elliptic shape and a random vector for the center. *Elliptic RHMs*, as presented in this paper, make use of a Gaussian distributed random vector that consists of the center and the shape.

All together, it highly depends on the particular application and requirements, which method for extended object tracking should be used.

6 Simulation Results

This section presents simulations results, which validate the insights about the two tracking methods worked out in the previous sections.

The presented simulations treat only non-moving targets because the main difference (of all discussed approaches) is the way the target extent is modeled. For concrete tracking examples, we refer the reader to [Koc08, FF09] and [BNH10].

6.1 Static Extended Object

The first example is a static extended object, where the extended object in fact has an elliptic shape. Moreover, the measurement sources are uniformly distributed on the ellipse surface, which is a natural assumption if there is no information about the target available. According to Section 5, both methods are capable of estimating the correct parameters of the ellipse in this case. In order to validate these results, we have performed three different simulations, where the measurement sources are sampled uniformly from the true ellipse. The number of measurements n_k in each scan k was Poisson-distributed with mean 5. The measurement noise is set to low ($\sigma = 20\,\mathrm{m}$), medium ($\sigma = 50\,\mathrm{m}$), and high ($\sigma = 75\,\mathrm{m}$) compared to the size of the ellipse (with diameters $300\,\mathrm{m}$ and $200\,\mathrm{m}$). The estimation results are shown Figure 3. There, it can be seen that both methods, i.e., SPD random matrices and *Elliptic RHMs*, provide accurate estimation results for all noise levels. The original random matrix approach [Koc08], which does not incorporate the measurement noise, overestimates the true ellipse.

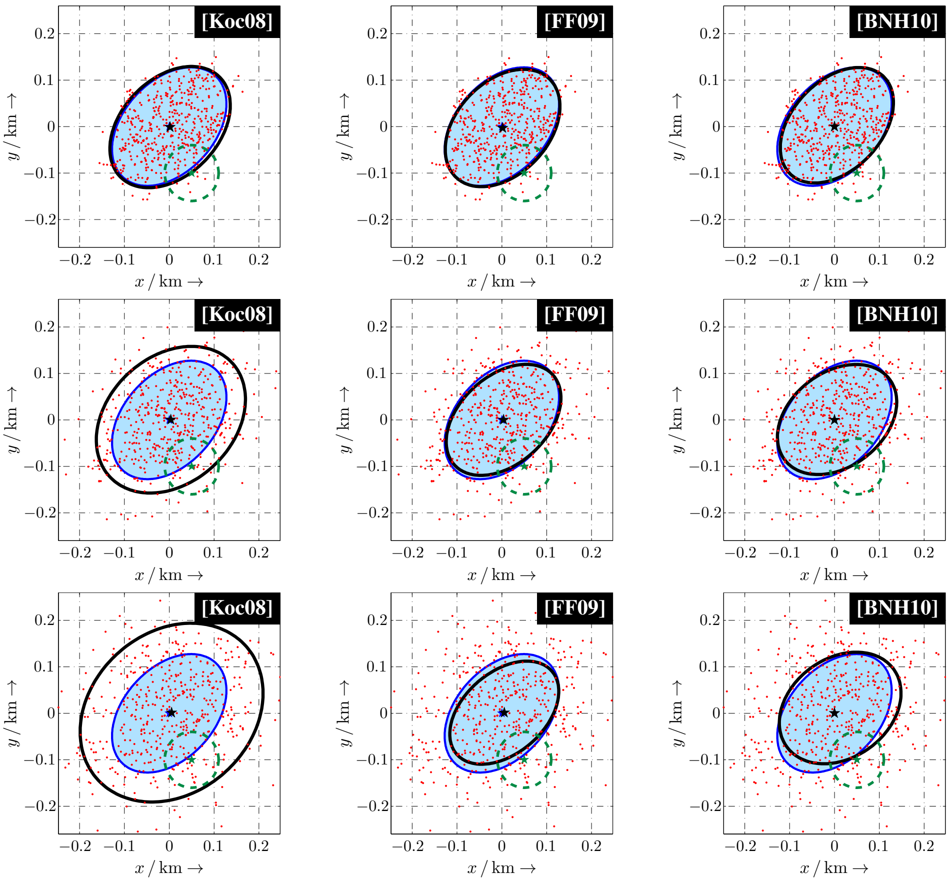

Figure 3: Simulation results for a static extended object, where the left column of figures summarizes the estimation results (black) of the original random matrix approach [Koc08] after 100 scans, the middle column the results of the alternative approach [FF09], and the right column the results of the *Elliptic RHM* approach [BNH10]. Shown are also the track initialization (green), the true ellipse (blue) and the accumulated measurements (red) of 100 scans with $\sigma = 20\,\mathrm{m}$ (top row), $\sigma = 50\,\mathrm{m}$ (middle row), and $\sigma = 75\,\mathrm{m}$ (bottom row).

6.2 Static Group of Point Targets

In the second example, the extent of a static group of 16 point targets is to be estimated. Each group member produces exactly one noisy measurement ($\sigma = 15\,\mathrm{m}$) at each time step. This example emphasizes that both tracking methods have a different interpretation of an ellipse. In fact, they estimate a different summarizing shape. While the tracking method using random matrices precisely estimates the sample mean and the scaled sample covariance matrix, the *Elliptic RHMs* aim at estimating the smallest enclosing ellipse of the target group. The methods estimate different quantities because the underlying model of the extended target is different. The estimation results are depicted in Figure 4. Note

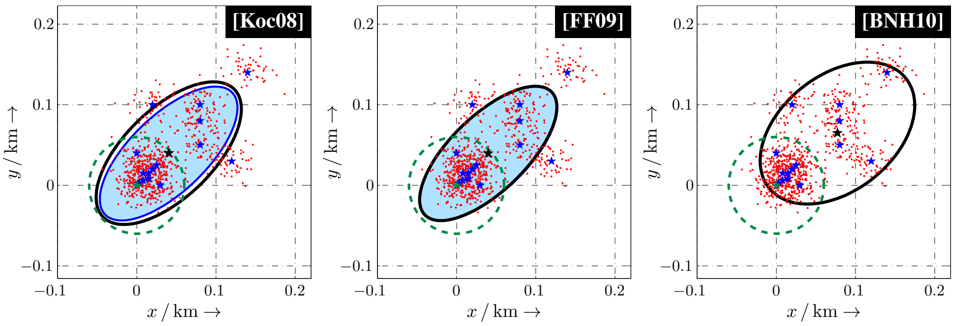

Figure 4: Simulation results for a static group of point targets, where the left part summarizes the estimation results (black) of the original random matrix approach [Koc08] after 50 scans, the middle part the results of the alternative approach [FF09], and the right part the results of the *Elliptic RHM* approach [BNH10]. Shown are also the scaled ellipse (light blue) of the sample covariance for the random matrix approaches, the true point target positions (blue), the accumulated measurements (red) of 50 scans, and the track initialization (green).

that the estimated ellipse of the advanced random matrix approach [FF09] (that accounts for measurement noise) as well as *Elliptic RHMs* do not depend on the measurement noise, as it is filtered out. However, the size of the estimated ellipse of the original random matrix approach [Koc08] (that neglects measurement errors) would increase with an increasing measurement error.

7 Conclusion and Future Work

In this work, a comparison of two tracking methods for extended targets has been given. The first method models the target extent with an SPD random matrix and the second one employs a so-called random hypersurface model. After a unified description of the two methods, we have given a detailed comparison of the different assumptions and properties of the methods. We have shown that the main difference lies in the interpretation of an elliptic extent. Both methods estimate an ellipse that summarizes the shape of the extended target. However, the quantity the estimated ellipse summarizes is different for both methods. Actually, the desired interpretation remains the user's choice because it highly depends on the particular application.

As already mentioned in the introduction, the explicit consideration of target extents is becoming more and more important in the development of modern tracking systems. However, extended object tracking is still a relatively new research area with a variety of open problems and challenges. In order to embed extended object tracking methods into a real-world tracking system, the proposed methods have to be extended to deal with clutter and multi-extended objects. In this context, it is also necessary to develop methods for splitting and merging tracks consisting of extended objects or groups of point targets.

In real-world scenarios, one typically has to deal with nonlinear measurement models for which both methods would have to be adapted. Finally, it would be interesting to develop methods that are able to estimate more complex shapes than ellipses.

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