Localization of DECT mobile phones based on a new nonlinear filtering technique

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ABSTRACT

In this paper, nonlinear Bayesian filtering techniques are applied to the localization of mobile radio communication devices. The application of this approach is demonstrated for the localization of DECT mobile telephones in a scenario with several base stations and a mobile handset. The received signal power, measured by the mobile handsets, is related to their position by nonlinear measurement equations. These consist of a deterministic part, modeling the received signal power as a function of the position, and a stochastic part, describing model errors and measurement noise. Additionally, user models are considered, which express knowledge about the motion of the user of the handset. The new Prior Density Splitting Mixture Estimator (PDSME), a Gaussian mixture filtering algorithm, significantly improves the localization quality compared to standard filtering techniques as the Extended Kalman Filter (EKF).

Keywords: Localization, Gaussian mixture densities, nonlinear filtering, Bayesian state estimation, nonlinear state estimation

1. INTRODUCTION

Localization of mobile radio communication devices is studied in various applications, which provide location-based services for users of mobile receivers. Therefore, developers of these services are interested in efficient approaches for the estimation of the position of the receiver, also called handset or mobile part. Typical examples are localization in WLAN, GSM or DECT networks. The estimation of the position should be based on information which is already available during normal operation of the phone for communication purposes.

A localization approach for GSM networks[1] relies on the location-dependant received signal power that is characteristic for the position of a receiver relative to the base stations (transmitters, fixed parts) and hence for the position of the user. During normal operation, the received signal power of all receivable transmitters is already measured by the mobile parts to permit a handover between different transmitters. Based on these measurements, localization algorithms can be developed to estimate the position of the mobile parts with respect to the transmitters. In various other publications,[2–6] applications of nonlinear state estimation techniques in the field of localization are presented.

In this paper, nonlinear filtering techniques are applied to the localization of DECT mobile telephones. To enable the localization of the mobile parts, an approximate stochastic model of the propagation of the radio waves or the received signal power is identified in the localization environment[7]. In principle, this information could be obtained from physical modeling of the propagation of the electromagnetic waves. However, in real-world environments, this is very complicated, since attenuation, reflexion, interference, and other phenomena yield very complicated distributions of the received signal power. As the exact parameters describing the electromagnetic properties of the environment where the handset is to be localized (localization environment) are almost always only partially known, physical modeling is prohibitive from a practical point of view. Therefore, a propagation

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model, relying on the measurement of the logarithmic received signal power in the localization environment, is preferred over deriving a detailed physical model theoretically.

For that purpose, a receiver is moved to various positions, e.g. gained by placing a grid on the localization area during preparatory work. At each position, the received signal power of the individual base stations is measured and stored in a map of the localization environment. Thus, distributions of the received signal power are gained for each base station.

Based on this model information, the position of the receiver can be estimated. In the localization phase, the easiest approach is the comparison of the received signal power of each receivable transmitter to the measured distributions of the received signal power, which have been stored during the model generation phase. The coordinates of the position with the smallest deviation between the model and the measured value are determined. Obviously, only a point estimate can be obtained by this next-neighbor-estimation. By this procedure, detailed information about the uncertainty of the estimated position cannot be obtained. Furthermore, the estimation quality is rapidly decreasing, if the distance between the reference points of the model is increased, e.g. if the precision of the grid is reduced during model generation. Additionally, the position estimate becomes worse, if less transmitters are receivable. Then, small variations of the measured received signal power or small variations of the positions of the receiver may lead to large variations in the estimated position. A further problem of this approach is that information about the motion of the user cannot be incorporated adequately in the estimation. Hence, by this simple procedure, localization is restricted to the determination of the position of the receiver by one single measurement of all receivable transmitters without taking into account previous measurements.

In this paper, a stochastic measurement model is identified in the model generation phase. This stochastic measurement model consists of an analytic, deterministic measurement function describing the logarithmic received signal power in terms of the position coordinates. Furthermore, a stochastic component is identified to take model uncertainties as well as measurement noise into account.

In contrast to the next-neighbor-estimation summarized above, a stochastic localization approach yields probability density functions representing the uncertainty of the estimated position. Besides considering the measurement of the received signal power, user models describing knowledge about the motion of a user of the receiver can furthermore be used to improve the localization quality. These user models e.g. express that the maximum "step length" of the user between the points of time corresponding to two different measurements of the received signal power is limited. Additional improvement of the estimation quality results from recursive position estimation by combining several subsequent measurements.

The stochastic localization approach presented in this paper leads to nonlinear, multi-dimensional measurement equations. The exact solution of this Bayesian filtering problem yields complicated, non-Gaussian probability densities describing the position estimate together with its uncertainty. These probability density functions are efficiently approximated by Gaussian mixture densities.[8,9] This approximation is done by the so-called Prior Density Splitting Mixture Estimator (PDSME), which has been developed to achieve an estimation quality, which can — in contrast to common filtering techniques for nonlinear systems (e.g. the Extended Kalman Filter[10]) — be specified by the user. However, specifying an upper bound for the computational effort by limiting the number of Gaussian mixture components may reduce the estimation quality. Furthermore, the representation of the densities by Gaussian mixtures enables recursive estimation for both nonlinear prediction and nonlinear filter steps.

In Section 2, the stochastic localization problem is formulated. In Section 3, the deterministic and stochastic components of the measurement model are explained. The application of a Bayesian filtering algorithm, based on a Gaussian mixture approximation of the exact probability densities using the new Prior Density Splitting Mixture Estimator (PDSME), is described in Section 4. Experimental results for the localization of radio communication devices are given in Section 5 for the localization of DECT receivers. The estimation quality of the PDSME is compared to the application of an Extended Kalman Filter (EKF). Finally, in Section 6 the paper is concluded.

2. PROBLEM FORMULATION

The localization of radio communication devices can be divided into two subproblems. The first subproblem is the identification of a measurement model — consisting of a deterministic and a stochastic part — in a model generation phase. Second, in the localization phase, the probability density of the estimated position is calculated by a Bayesian filter step.

The measurement model

$$\underline{\hat{y}}_k = \begin{bmatrix} \hat{y}_{k,1} \\ \vdots \\ \hat{y}_{k,N} \end{bmatrix} = \begin{bmatrix} h_1(\underline{x}_k) \\ \vdots \\ h_N(\underline{x}_k) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} = \underline{h}(\underline{x}_k) + \underline{v}_k \tag{1}$$

describes the measurement of the logarithmic received signal power

$$\hat{y}_{k,\zeta} = 10 \cdot \log_{10} \left(\frac{P_{k,\zeta}}{1\text{mW}} \right)$$

of the ζ-th transmitter, $\zeta = 1, \ldots, N$, as a deterministic, nonlinear function $h_\zeta(\underline{x}_k)$ of the position coordinates $\underline{x}_k = \begin{bmatrix} x_{1,k} & x_{2,k} \end{bmatrix}^T$ for each of the N different transmitters. The stochastic part $\underline{v}_k$ of the measurement model is represented by an additive uncertainty. Additive uncertainties in the measurement model of the logarithmic received signal power correspond to multiplicative uncertainties of the signal power $P_{k,\zeta}$, which are caused by the influence of uncertain attenuation. Each transmitter can be identified by a unique ID, that is transmitted by the base stations during communication. Therefore, the estimation problem is simplified significantly, because each measured value can directly be assigned to the corresponding nonlinear measurement equation.

In the localization phase, nonlinear filtering algorithms are applied in each time-step k. Even without additive uncertainties $\underline{v}_k$, the implicit, nonlinear relations between the vector of the measured logarithmic received signal power $\underline{\hat{y}}_k$ of the transmitters and the position $\underline{x}_k$ of the receiver can only be solved numerically, e.g. by a least-squares approach. Considering uncertainties, the measurement equations of the logarithmic received signal power are used to update the estimated position in a Bayesian filter step.

The localization approach presented in this paper is based on the approximation of the exact solution of the Bayesian filter step by Gaussian mixture densities

$$f_x^p(\underline{x}_k) = \sum_{i=1}^{L} \omega_{x,k}^{p,i} \frac{\exp\left(-\frac{1}{2} \left\| \underline{x}_k - \underline{\mu}_{x,k}^{p,i} \right\|^2_{\left(\mathbf{C}_{xx,k}^{p,i}\right)^{-1}} \right)}{\sqrt{(2\pi)^2 \left| \mathbf{C}_{xx,k}^{p,i} \right|}}$$

with L components, defined by non-negative weighting factors $\omega_{x,k}^{p,i}$, expected values $\underline{\mu}_{x,k}^{p,i}$ and covariances $\mathbf{C}_{xx,k}^{p,i}$. If a measured value $\hat{y}_{k,\zeta}$ is available, the exact solution of the Bayesian filter step[11, 12] is given by

$$f_x^e(\underline{x}_k | \hat{y}_{k,\zeta}) = c\, f_x^p(\underline{x}_k)\ f_{v,\zeta}(\hat{y}_{k,\zeta} - h_\zeta(\underline{x}_k)) \quad ,$$

with the nonlinear measurement function $h_\zeta(\underline{x}_k)$, the additive uncertainty's density function $f_{v,\zeta}(v_\zeta)$ and a normalization constant c. Prior knowledge about the estimated position is represented by the density function $f_x^p(\underline{x}_k)$. For each new measurement the preceding posterior density $f_x^e(\underline{x}_k | \hat{y}_{k,\zeta})$ is interpreted as the new prior density, so that the estimated position can be updated recursively.

Analogously to the Bayesian filter step, a prediction step, which is considered to describe nonlinear user models

$$\underline{x}_{k+1} = \underline{a}_k(\underline{x}_k) + \underline{w}_k \quad , \tag{2}$$

is also treated by approximating the exact densities by Gaussian mixtures. The nonlinear function $\underline{a}_k(\underline{x}_k)$ is a deterministic model of the motion of the user. Uncertainties are again expressed by additive noise $\underline{w}_k$.

3. STOCHASTIC MODELING APPROACH

In this Section, a measurement model is identified by measuring the logarithmic received signal power of each transmitter on a grid covering the localization area. The deterministic component $\underline{h}(\underline{x}_k) \in \mathbb{R}^N$ and the stochastic component $\underline{v}_k$ of the measurement model (1) have to be identified before localization. The deterministic, analytic part $\underline{h}(\underline{x}_k)$ is a measurement equation that describes the received power as a function of the position coordinates $\underline{x}_k$. The stochastic part $\underline{v}_k$ is a model of the uncertainties of the deterministic component. These uncertainties consist of both spatial uncertainties corresponding to model errors and temporal measurement noise.

3.1. Deterministic measurement model of the logarithmic received signal power

In Fig. 1 it can be seen that the decrease of the logarithmic received signal power over a large-scale distance of several meters is approximately linear. In other publications, similar assumptions on the measurement model are either called "linear-loss-model"[13] or "linear-slope-model"[7]. Mathematically, for 2-dimensional position coordinates $\underline{x}_k$ a direction-dependant linear decrease of the logarithmic received power is described by N independent measurement equations

$$h_\zeta(\underline{x}_k) = -\sqrt{\left\|\underline{x}_k - \underline{m}_\zeta\right\|^2_{\mathbf{P}_\zeta^{-1}}} + \Delta_\zeta \quad . \tag{3}$$

The parameters $\underline{m}_\zeta$ and $\mathbf{P}_\zeta$ of the positive semidefinite quadratic form $\left\|\underline{x}_k - \underline{m}_\zeta\right\|^2_{\mathbf{P}_\zeta^{-1}}$ and the additive offset Δ_ζ have to be identified for each transmitter $\zeta = 1, \ldots, N$.

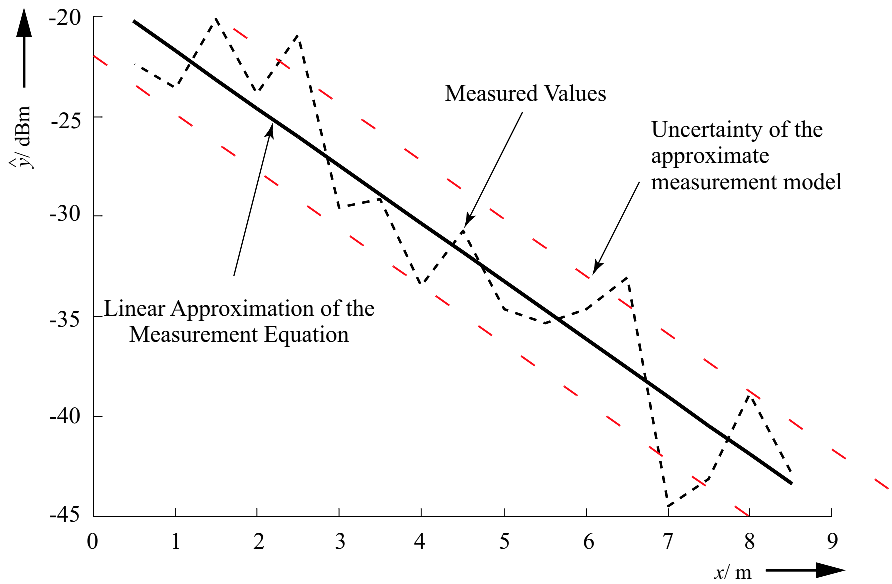

Figure 1. Assumed linear approximation of the measured logarithmic received signal power and uncertainties of the approximate measurement model caused by deviations between the measurements and the assumed measurement model.

For two-dimensional position coordinates $\underline{x}_k = \begin{bmatrix} x_{1,k} & x_{2,k} \end{bmatrix}^T$, the $6N$ parameters $\underline{m}_\zeta$, $\mathbf{P}_\zeta$ and Δ_ζ are determined by a least-squares-method, which minimizes the deviation between the approximated measurement model and the logarithmic received power on the grid points measured during the model generation phase. To reduce temporal measurement noise, the mean of several measurements at each grid point is used to calculate the average of the measured values.

3.2. Stochastic modeling of uncertainties

The stochastic uncertainty model has to consider both deviations between the approximated model derived in Subsection 3.1 and the true distribution of the logarithmic received signal power measured on the grid, and temporal measurement noise.

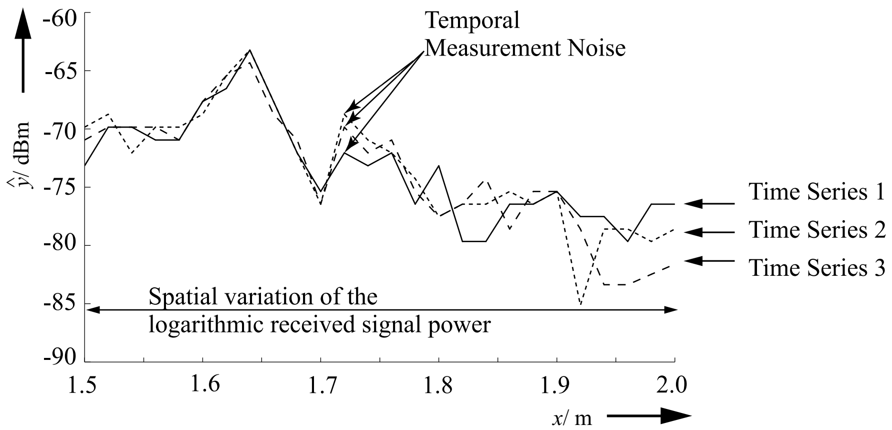

Figure 2. Spatial variations of the logarithmic received signal power over short distances and temporal measurement noise (three different time series).

3.2.1. Model uncertainties

In this localization approach, the deviation between the true logarithmic received signal power and the deterministic measurement model $\underline{h}(\underline{x}_k)$ is described by a Gaussian noise density with mean $\mu_{v,\zeta}^{\langle 1 \rangle}$ and standard deviation $\sigma_{v,\zeta}^{\langle 1 \rangle}$ for each transmitter. This Gaussian uncertainty is a representation for the approximation error of the deterministic component of the measurement model over the whole localization environment. First, it represents the mean deviation over the localization environment between the model and the true received power, because of incorrect assumptions for the deterministic part of the measurement equation. The uncertainties resulting from large-scale deviations between the true received signal power and the measurement model are depicted in Fig. 1. Second, measurements have also shown that there exist local deviations, that might be caused by reflexion, non-homogenous propagation of the radio waves and interference. These spatial variations of the logarithmic received signal power can be seen in Fig. 2, where measurements have been collected with a distance of 2 cm.

3.2.2. Measurement noise

In addition to spatial variations of the received signal power, temporal measurement noise can be determined by analyzing a series of several different measurements at a fixed position. This temporal measurement noise is also shown in Fig. 2, for three different measurements of the logarithmic received signal power at each measurement position. This noise is again approximated by a Gaussian density with mean $\mu_{v,\zeta}^{\langle 2 \rangle}$ and standard deviation $\sigma_{v,\zeta}^{\langle 2 \rangle}$.

3.2.3. Combination of model uncertainties and measurement noise

To obtain a simple model comprising both uncertainties described in 3.2.1 and 3.2.2, it is further assumed, that both uncertainties are independent. Therefore, they can be modeled by a single Gaussian density $f_{v,\zeta}(v_\zeta)$ for each transmitter $\zeta = 1, \ldots, N$, defined by the mean

$$\mu_{v,\zeta} = \mu_{v,\zeta}^{\langle 1 \rangle} + \mu_{v,\zeta}^{\langle 2 \rangle}$$

and the standard deviation

$$\sigma_{v,\zeta} = \sqrt{\left(\sigma_{v,\zeta}^{\langle 1 \rangle}\right)^2 + \left(\sigma_{v,\zeta}^{\langle 2 \rangle}\right)^2} \; .$$

This model not only implies independent uncertainties, but it is also assumed that both uncertainties can be described without considering any position dependency.

4. FILTERING ALGORITHM

In this Section, a brief overview of the Prior Density Splitting Mixture Estimator (PDSME) used for the localization of radio communication devices is given. Additionally, an adaptation of the measurement equations $\underline{h}(\underline{x}_k)$ derived in Section 3 is introduced to simplify the calculation of the PDSME. Furthermore, a prediction step for a simple user model is presented.

4.1. The PDSME measurement update

The PDSME algorithm for the localization of radio communication devices presented in this paper is based on the calculation of a linearized measurement update for Gaussian mixture densities. The measurement update step of this filtering algorithm is shown in a block diagram in the upper part of Fig. 3. Splitting is based on the calculation of the linearization error

$$\mathcal{D}_2\left(\overline{f}_x^{e,i} \| f_x^{e,i}\right) = \int\limits_{\mathbb{R}^2} \overline{f}_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta}) \left(\ln\left(\frac{\overline{f}_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})}{f_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})}\right)\right)^2 d\underline{x}_k \quad (4)$$

for each component of the posterior Gaussian mixture density. This criterion is very similar to the Kullback-Leibler distance[14]

$$\mathcal{D}\left(\overline{f}_x^{e,i} \| f_x^{e,i}\right) = \int\limits_{\mathbb{R}^2} \overline{f}_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta}) \ln\left(\frac{\overline{f}_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})}{f_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})}\right) d\underline{x}_k$$

between the exact posterior density $f_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})$ and its approximation $\overline{f}_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})$ by replacing the nonlinear measurement equation $\underline{h}(\underline{x}_k)$ by its linearization $\overline{\underline{h}}(\underline{x}_k)$ at the mean of the i-th component of the prior density function. Calculating the linearization error (4), the prior Gaussian mixture components which contribute most to the approximation error of the posterior density are identified. To reduce this linearization error, these Gaussian mixture components are replaced by splitting them into several mixture components with smaller covariances using splitting libraries, which have been optimized off-line (see Fig. 4 in[15]).

After this "analytic resampling" of the prior density, the filter step can be calculated by a bank of EKFs, linearizing the measurement equation at the mean of each component of the Gaussian mixture representation of the prior density.

Afterwards, a merging step reduces redundancy by combining several Gaussian mixture components to a single Gaussian with negligible approximation error.

4.2. Adaptation of the measurement model for simplifications of the linearization error criterion

For Gaussian measurement noise, the linearization error (4) can be calculated analytically as a linear combination of moments of the density $\overline{f}_x^{e,i}(\underline{x}_k|\hat{y}_{k,\zeta})$ for polynomial measurement equations $h_\zeta(\underline{x}_k)$. Therefore, in this Subsection, an adaptation of the measurement equation (3) is derived. After some algebraic conversions, the squared measurement equation

$$\underbrace{(\hat{y}_{k,\zeta} - \Delta_\zeta)^2}_{=\hat{z}_{k,\zeta}} = \left\|\underline{x}_k - \underline{m}_\zeta\right\|^2_{\mathbf{P}_\zeta^{-1}} + \underbrace{\left(2\left(\hat{y}_{k,\zeta} - \Delta_\zeta\right) v_\zeta - v_\zeta^2\right)}_{=\tilde{v}_\zeta} \quad (5)$$

can be re-written as a polynomial function with a modified "measured value" $\hat{z}_{k,\zeta}$ and the transformed uncertainty $\tilde{v}_\zeta$. Because of the nonlinear transformation of the random variable v_ζ, the probability density function $f_{\tilde{v},\zeta}(\tilde{v}_\zeta)$ is no longer Gaussian. In the localization experiment in Section 5, the exact first and second order moments of $\tilde{v}_\zeta$ are computed to determine a Gaussian approximation of $f_{\tilde{v},\zeta}(\tilde{v}_\zeta)$. Note, that the moments of $\tilde{v}_\zeta$ are depending upon the measured value $\hat{y}_{k,\zeta}$. Therefore, they have to be re-computed for each new measurement of the logarithmic received signal power and are not time-invariant as the parameters $\mu_{v,\zeta}$ and $\sigma_{v,\zeta}$ described in Section 3.

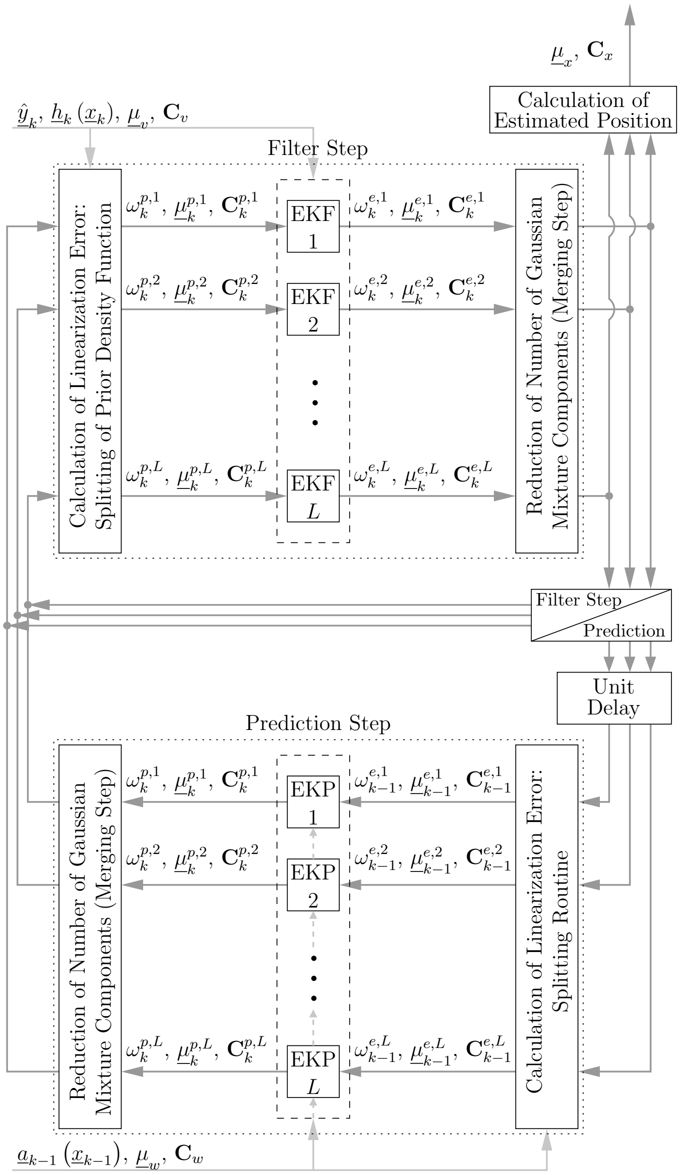

Figure 3. Overview of the PDSME algorithm: The PDSME consists of a linearized filter step (upper part) and linearized prediction step (lower part).

4.3. User modeling by PDSME prediction step

Similarly to the filter step, the PDSME can also be applied to the calculation of nonlinear prediction steps. Analogously to nonlinear filter steps, the calculation of an approximated prediction step also consists of the evaluation of a linearization error, a bank of linearized prediction steps and the reduction of the number of Gaussian mixture components in a merging step (see the lower part of Fig. 3). In this paper, only a linear user model is considered. Therefore, the prediction step can be calculated analytically, since the posterior density has been approximated by a Gaussian mixture density in the filter step.

The prediction model consists of the linear state equation

$$\underline{x}_{k+1} = \underline{x}_k + \underline{w}_k \quad ,$$

where the mean $\underline{\mu}_w$ of the additive system noise $\underline{w}_k$ represents knowledge about possible *directions* and *mean step lengths* of the user's movement. The covariance matrix $\mathbf{C}_w$ of $\underline{w}_k$ specifies an estimate for the distribution of the user's *step lengths*. For each component $i = 1, \ldots, L$ of the Gaussian mixture density, the predicted Gaussian mixture component is then described by the mean

$$\underline{\mu}_{k+1}^i = \underline{\mu}_k^i + \underline{\mu}_w$$

and the covariance

$$\mathbf{C}_{k+1}^i = \mathbf{C}_k^i + \mathbf{C}_w \quad .$$

The estimated position can then be calculated as a weighted superposition

$$\frac{\sum_{i=1}^{L} \omega_{k+1}^i \underline{\mu}_{k+1}^i}{\sum_{i=1}^{L} \omega_{k+1}^i}$$

of the means of all Gaussian mixture components.

5. LOCALIZATION EXPERIMENT

In this Section, a real-world localization experiment is presented for the validation of the described localization approach for DECT mobile telephones. In the Subsections 5.2 and 5.3, the superior performance of the PDSME compared to the Extended Kalman Filter (EKF),[16] a widely used standard approach for the state estimation of nonlinear systems, is shown.

5.1. Evaluation scenario

In this localization experiment, $N = 10$ transmitters have been placed in an indoor area of approximately $30\,\mathrm{m} \times 30\,\mathrm{m}$ in one floor of a building. In the model generation phase, the logarithmic received signal power of each transmitter has been measured on a grid with 1 m distance between grid points. $N = 10$ measurement equations $h_\zeta(\underline{x}_k)$, altogether consisting of 60 parameters for the deterministic components and 20 parameters for the stochastic components $\underline{v}_k$, have been identified. Except for a few areas, influenced by high attenuation of the radio waves because of ferroconcrete walls, the assumed model is an appropriate approximation of the distribution of the received signal power.

The initial probability density of the position is chosen as a Gaussian density with the initial mean

$$\underline{\mu}_0^p = \begin{bmatrix} 15\,\mathrm{m} \\ 15\,\mathrm{m} \end{bmatrix}$$

and the initial covariance

$$\mathbf{C}_0^p = \begin{bmatrix} 15^2 & 0 \\ 0 & 15^2 \end{bmatrix} \mathrm{m}^2 \quad ,$$

i.e., almost no prior knowledge about the position is available.

In the localization phase, a measurement of each receivable transmitter is collected along a line as shown in Fig. 4, with a distance of $\Delta\tau = 0.5$m between the true measurement positions $P_1, P_2, \ldots, P_{17}$. The position coordinates of the measurements have been determined to compare the ground truth to the estimated positions. Neither in the localization approach by the PDSME nor by the EKF, the exact measurement positions have been used for the localization of the receiver. After the measurement update has been calculated for each receivable transmitter in a fixed position P_i, $i = 1, \ldots, 17$, a prediction step according to Subsection 4.3 has been calculated. In this example, the simple user model is defined by the mean

$$\underline{\mu}_w = \underline{0}\,\mathrm{m}$$

and the covariance

$$\mathbf{C}_w = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathrm{m}^2 \quad ,$$

i.e., no knowledge about a preferred direction of the motion of the user is available. Only the distribution of the user's step lengths is represented by the covariance matrix $\mathbf{C}_w$.

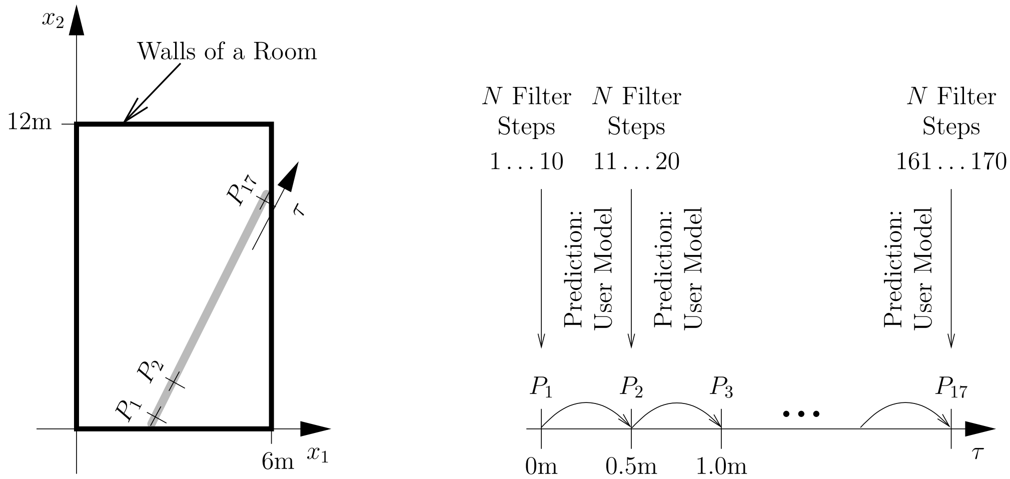

Figure 4. Evaluation scenario of the localization experiment.

5.2. Extended Kalman Filter (EKF)

Using the EKF, the measurement equation (5) is linearized at the mean of the prior density function. In Fig. 5, the results of the EKF are presented for the first and the 170th filter step. In each filter step, the measurement of the logarithmic received signal power of a single transmitter is used to update the estimated position.

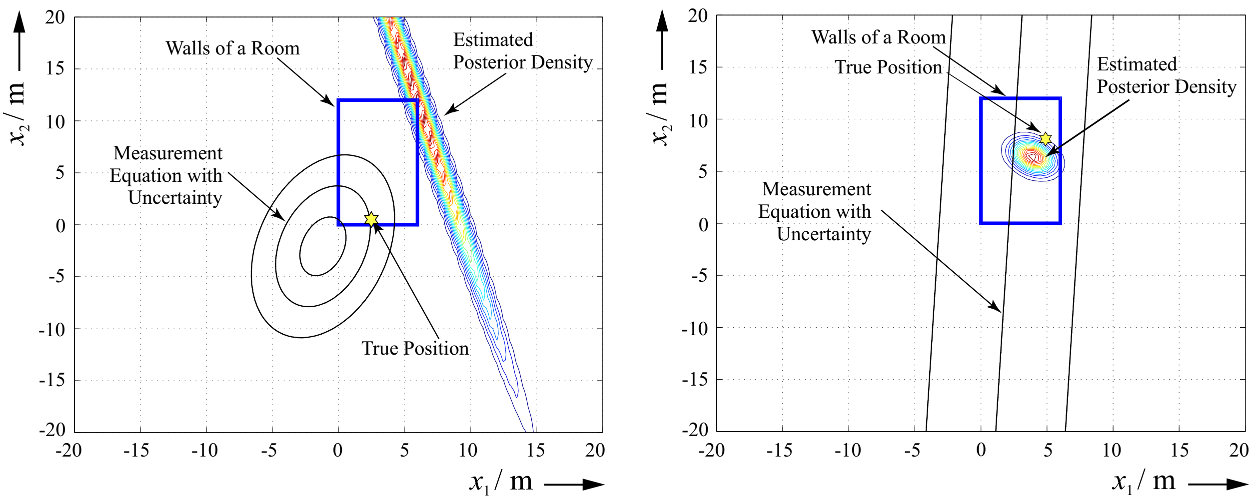

Figure 5. Localization with the EKF: First filter step (left) and 170th filter step (right).

It can be noticed, that there is a significant estimation error after the first filter step. The true position of the receiver is not within the support of the estimated posterior density function depicted by its contour plot. Furthermore, there is no intersection between the true measurement equation and the estimated density. Therefore, it is not possible to apply data validation techniques to find out whether a measured value can be "explained" by the estimated density.

5.3. Prior Density Splitting Mixture Estimator (PDSME)

In Fig. 6, the posterior densities estimated by the PDSME algorithm are also shown for the first and 170th filter step. Obviously, the approximation of the non-Gaussian posterior density in the first filter step, which is very close to the numerically calculated optimal Bayesian solution of the filter step, is much better than in the case of the EKF. Hence, data validation techniques can now be successfully applied.

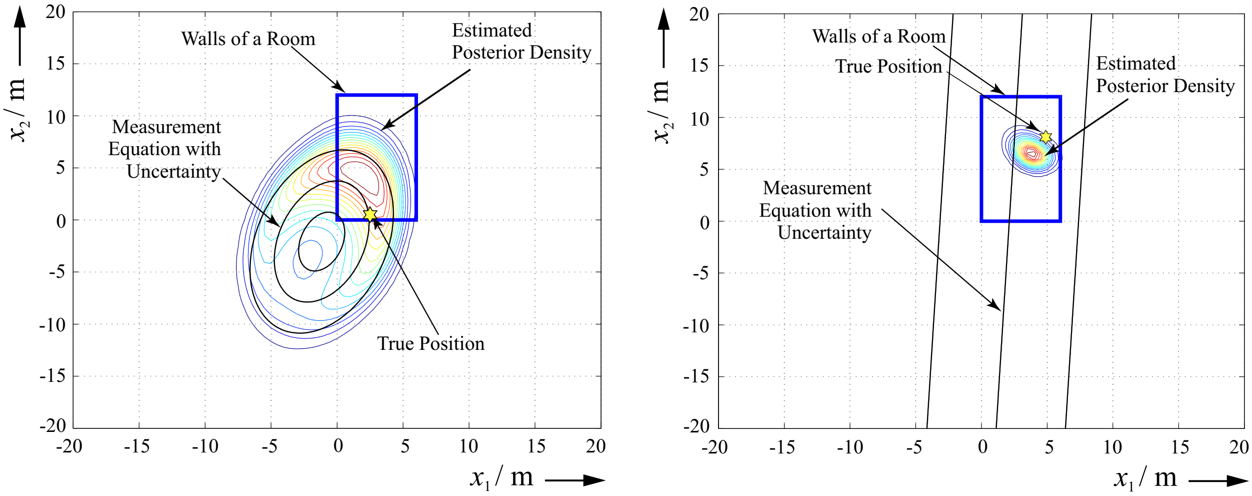

Figure 6. Localization with the PDSME: First filter step (left) and 170th filter step (right).

In Fig. 7, the expected values of the posterior density functions calculated by the EKF and the PDSME are compared to the true position. Comparing this figure to Fig. 8, it can be seen, that for almost linear filtering problems, i.e., if the covariance of the estimated position is small compared to the nonlinearity of the measurement equation after several filter steps, the EKF and the PDSME yield almost the same results. In these cases, the PDSME only uses a moderate number of Gaussian mixture components, whereas for strong nonlinearities at the beginning of the localization experiment a higher number of approximation components is necessary to reduce the estimation error. The superior performance of the PDSME compared to the EKF is also shown by the average estimation error

$$\frac{1}{N_F} \sum_{k=1}^{N_F} \sqrt{\left\| \underline{x}_k^{true} - \underline{x}_k^{estimated} \right\|_2^2}$$

over the $N_F = 170$ filter steps, which is 3.30 m for the EKF and 1.22 m for the PDSME.

6. CONCLUSIONS

In this paper, a stochastic approach for the localization of radio communication devices has been presented, which is based on measuring the logarithmic signal power of the receivable transmitters by a mobile part. A stochastic measurement model, consisting of a deterministic and a stochastic component has been identified for each transmitter. This measurement model has been used for the estimation of the position of the receiver by a novel Gaussian mixture estimator, which is based on splitting the prior density according to a linearization error criterion. This criterion is very similar to the Kullback-Leibler distance between the true and the approximated posterior density, calculated by a linearization of the measurement equation. Applying this estimation technique to the localization of DECT mobile telephones, significant improvements of the estimation quality can be achieved, if the PDSME is used instead of standard approaches like the EKF. Further improvement of the localization quality can be achieved by identifying better deterministic measurement models and more precise characterizations of the measurement noise, which do not assume independence between the different uncertainties mentioned in this paper.

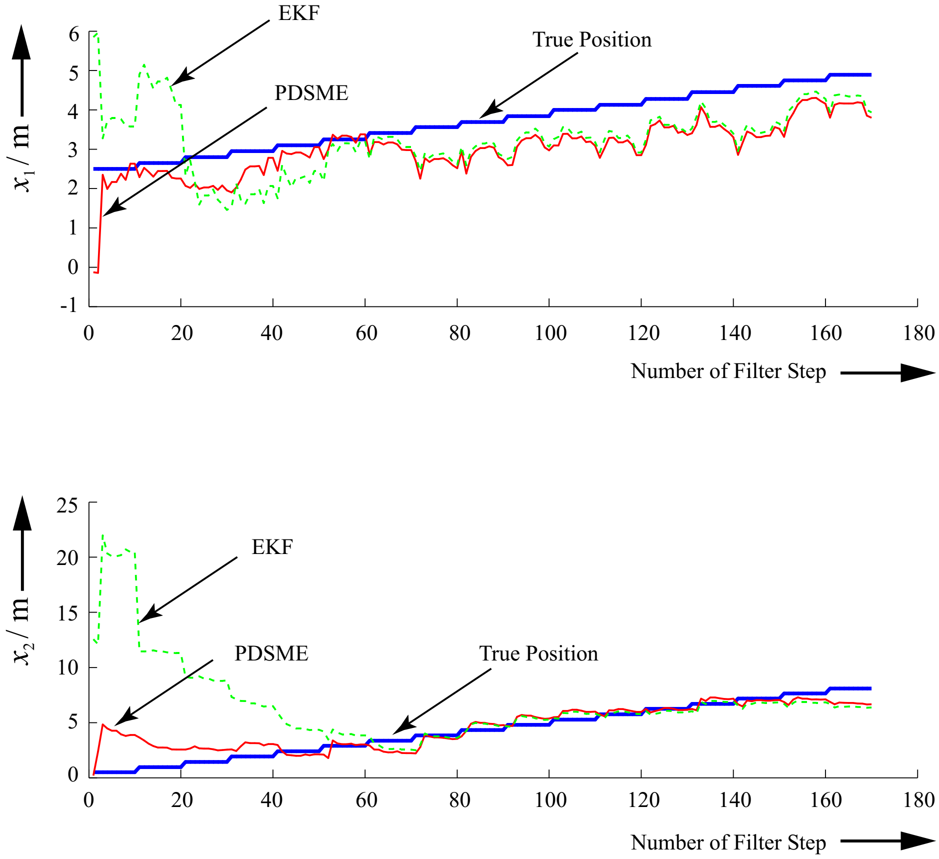

Figure 7. Comparison of the expected values of the estimated position calculated by the EKF (dashed lines) and the PDSME (thin solid lines) to the exact position (thick solid lines) for the x_1 and x_2 coordinates.

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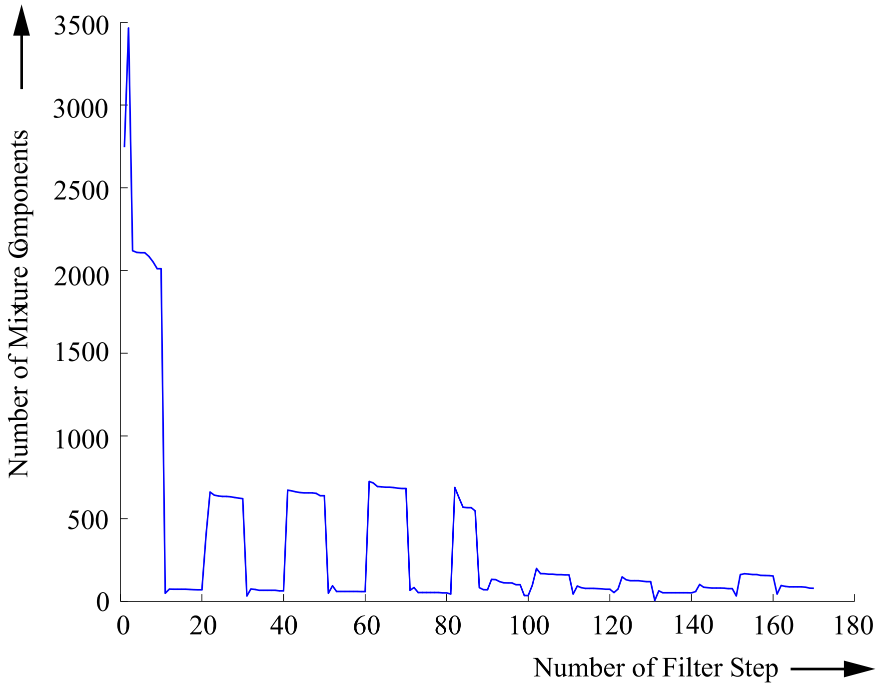

Figure 8. Number of Gaussian mixture components used by the PDSME.

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